

# Certified Unsolvability in Classical Planning

## 2. Applications

Salomé Eriksson   Gabriele Röger   Malte Helmert

University of Basel, Switzerland

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# Certified Unsolvability in Classical Planning

## 2. Applications

### 2.1 Certificate Structure

### 2.2 Blind Search

### 2.3 Heuristic Search

### 2.4 $h^2$ Preprocessor

### 2.5 Recap

# Certificate Structure

## First look

on Github: <https://github.com/salome-eriksson/helve>

A certificate consists of the following files:

- ▶ task description
  - ▶ limited to STRIPS
    - STRIPS with negation coming soon™
  - ▶ variable and action IDs (according to occurrence in list)
- ▶ certificate
  - ▶ **state sets:** e ID type description  
e 0 h p cnf 3 2 2 -1 0
  - ▶ **action sets:** a ID type description  
a 0 u 2 4 5 6
  - ▶ **statements:** k ID type set-ID(s) justification  
premises k 0 d 7 sd 5 4
- ▶ (BDD descriptions)

(detailed explanation in README.md of github repository)

## Certificate File

### Example

what we have seen so far:

#	statement	justification
(0)	$\emptyset$ dead	<b>ED</b>
(1)	$\{\mathbf{I}\} \sqsubseteq \text{states}(\neg a \vee \neg b)$	<b>B1</b>
(2)	$S_{\neg a \vee \neg b}[\mathbf{A}] \sqsubseteq S_{\neg a \vee \neg b} \cup \emptyset$	<b>B2</b>
(3)	$S_{\neg a \vee \neg b}$ dead	<b>PI</b> with (3),(1) and (2)

...

the corresponding certificate file:

1	e 0 c e	7	e 3 p 2 0
2	e 1 c i	8	e 4 u 2 0
3	a 0 a	9	k 2 s 3 4 b2
4	k 0 d 0 ed	10	e 5 n 2
5	e 2 h p cnf 2 1 1 2 0	11	k 3 d 5 pi 2 0 1
6	k 1 s 1 2 b1	...	

→ Demo

## Blind Search

## High-Level Certificate

Blind search explores all **reachable states**  $\mathcal{R}^\Pi$ .

→ Recall completeness proof:

### Forward Blind Search Certificate

#	statement	justification
(0)	$\emptyset$ dead	<b>ED</b>
(1)	$\mathcal{R}^\Pi[\mathbf{A}] \sqsubseteq \mathcal{R}^\Pi \cup \emptyset$	<b>B2</b>
(2)	$\mathcal{R}^\Pi \cap \mathbf{S}_G \sqsubseteq \emptyset$	<b>B1</b>
(3)	$\mathcal{R}^\Pi \cap \mathbf{S}_G$ dead	<b>SD</b> with (0) and (2)
(4)	$\mathcal{R}^\Pi$ dead	<b>PG</b> with (1), (0) and (3)
(5)	$\{\mathbf{I}\} \sqsubseteq \mathcal{R}^\Pi$	<b>B1</b>
(6)	$\{\mathbf{I}\}$ dead	<b>SD</b> with (4) and (5)
(7)	unsolvable	<b>CI</b> with (6)

For backwards search, show  $\{\mathbf{I}\} \sqsubseteq \overline{\mathcal{R}_B^\Pi}$  and  $[\mathbf{A}]\mathcal{R}_B^\Pi \sqsubseteq \mathcal{R}_B^\Pi \cup \emptyset$ , deduce  $\mathcal{R}_B^\Pi$  dead (rule **RI**), and from  $\mathbf{S}_G \sqsubseteq \mathcal{R}_B^\Pi$  show  $\mathbf{S}_G$  dead.

## Translation to Certificate File

1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \sqsubseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \sqsubseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead
13	k 4 d 3 pg 1 0 3	(4) $S_3 (= \mathcal{R}^\Pi)$ dead
14	k 5 s 1 3 b1	(5) $S_1 (= \{\mathbf{I}\}) \sqsubseteq S_3 (= \mathcal{R}^\Pi)$
15	k 6 d 1 sd 5 4	(6) $S_1 (= \{\mathbf{I}\})$ dead
16	k 7 u ci 6	(7) unsolvable

## Implementation Details

Depending on the concrete algorithm, some implementation details affect performance:

- ▶ formalism for  $\mathcal{R}^\Pi$ 
  - ▶ symbolic search: BDD
    - overhead if no singular closed BDD!
  - ▶ explicit search: explicit enumeration or BDD
    - fast generation vs fast verification
- ▶ when to build the certificate
  - ▶ during search: unnecessary overhead for solvable problems
  - ▶ at the end: more overhead (iterate over entire closed list), but also more localized

## Heuristic Search

## Idea

- 1 Each dead-end is dead.
- 2 The set of expanded states contains no goal state.
- 3 The set of expanded states can only reach itself and dead-ends.
- 4 → the set of expanded states is dead.
- 5 The initial state is in the set of expanded states.
- 6 → The initial state is dead and the task unsolvable.

## High-Level Certificate

#	statement	justification
(1)	$\emptyset$ dead	
(2)	$\{d_1\}$ dead	(3) $\{d_2\}$ dead ... (4) $\{d_n\}$ dead
(5)	$\{d_1\} \cup \{d_2\}$ dead	<b>UD</b> with (2) and (3)
...	...	...
(6)	$\bigcup_{1 \leq i \leq n} d_i$ dead	<b>UD</b> with ...
(7)	$S_D \subseteq \bigcup_{1 \leq i \leq n} d_i$	<b>B1</b>
(8)	$S_D$ dead	<b>SD</b> with (6) and (7)
(9)	$S_{\text{exp}}[\mathbf{A}] \subseteq S_{\text{exp}} \cup S_D$	<b>B2</b>
(10)	$S_{\text{exp}} \cap S_G \subseteq \emptyset$	<b>B1</b>
(11)	$S_{\text{exp}} \cap S_G$ dead	<b>SD</b> with (1) and (10)
(12)	$S_{\text{exp}}$ dead	<b>PG</b> with (9), (8) and (11)
(13)	$\{I\} \subseteq S_{\text{exp}}$	<b>B1</b>
(14)	$\{I\}$ dead	<b>SD</b> with (12) and (13)
(15)	task unsolvable	<b>CI</b> with (14)

## Bridging Representations

Statements “ $\{d_i\}$  dead” might use **different representations**.

- 1 Show  $\{d_i\}_{\text{explicit}} \sqsubseteq \{d_i\}_{\mathbf{R}}$  (basic statement **B4**)

and then either

- 2a build  $(S_{\text{exp}})_{\text{explicit}}$ , and
- 3a show  $(S_{\text{exp}})_{\text{explicit}} \sqsubseteq (S_{\text{exp}})_{\text{explicit}} \cup \bigcup \{d_i\}_{\text{explicit}}$  (**B2**).

or

- 2b build  $(S_D)_{\text{explicit}}$ ,  $(S_D)_{\text{BDD}}$  and  $(S_{\text{exp}})_{\text{BDD}}$ ,
- 3b show  $(S_D)_{\text{explicit}} \sqsubseteq \bigcup \{d_i\}_{\text{explicit}}$  (**B2**),
- 4b show  $(S_D)_{\text{BDD}} \sqsubseteq (S_D)_{\text{explicit}}$  (**B4**), and
- 5b show  $(S_{\text{exp}})_{\text{BDD}} \sqsubseteq (S_{\text{exp}})_{\text{BDD}} \cup (S_D)_{\text{BDD}}$  (**B2**).

→ tradeoff efficient generation vs efficient verification

## Delete Relaxation

$h^{\text{max}}$  dead-end

$h^{\text{max}}(s) = \infty \leftrightarrow$  some  $g \in \mathbf{G}$  relaxed unreachable

Consider  $R_u^+(s) = \{v \mid v \text{ relaxed unreachable from } s\}$  and

$\varphi = \bigwedge_{v \in R_u^+(s)} \neg v$ .

- ▶ We can't reach any  $s'$  containing any  $v \in R_u^+(s)$ :  $S_\varphi[\mathbf{A}] \subseteq S_\varphi$
- ▶ All states satisfying  $\varphi$  do not satisfy  $g$ :  $S_\varphi \cap \mathbf{S}_G = \emptyset$
- ▶ State  $s$  satisfies  $\varphi$ :  $\{s\} \subseteq S_\varphi$

→ Show that  $S_\varphi$  is dead (**PG**) and thus  $s$  is dead (**SD**).

We can choose between different **representations**:

BDD, Horn formula, 2CNF formula, explicit (over  $R_u^+(s)$ )

## $h^m$ & Clause-Learning State Space Search

$h^m$  dead-ends:

- ▶ same concept as  $h^{\text{max}}$ :  $\varphi = \bigwedge_{t \in R_u^m(s)} \bigvee_{v \in t} \neg v$ , where  $R_u^m(s)$  are the tuples unreachable from  $s$ .
- ▶ representation: Horn formulas (or 2CNF formulas for  $m = 2$ )  
→ **BDDs not suited** [Edelkamp & Kissmann 2011]

Clause-Learning State Space Search [Steinmetz & Hoffmann (2017)]:

- ▶ uses  $h^C$  → same concept (again)
- ▶ can be refined to detect **I** as dead-end → compact certificate
- ▶ uses additional source for mutexes  
→ **Integrate additional information into certificate!**

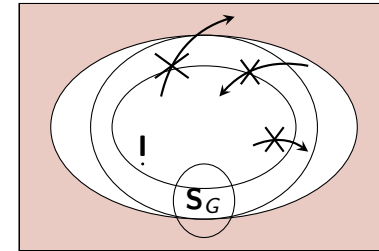
## Other heuristics

- ▶  $h^{\text{max}}$  approach covers **all** delete-relaxation heuristics ( $h^{\text{LM-Cut}}$ , landmarks based on delete relaxation, ...)
- ▶ Merge & Shrink:
  - ▶ transformation from Merge & Shrink representation to ADD [Helmert et al. 2014] and extract  $\infty$ -paths to BDD  
→ limited to **linear merge strategies** [Helmert et al. 2015]
  - ▶ one set for **all dead-ends**  
→ certificate more compact
  - ▶ implementation detail: unreachable and dead-end states merged  
→ **disable for certificate generation**

# $h^2$ Preprocessor

## Algorithm Overview

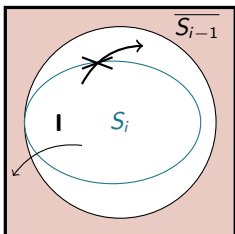
- ▶ introduced in [Alcázar & Torralba (2015)]
- ▶ preprocessing step that simplifies planning task
- ▶ used in many IPC planners
- ▶ **incremental**  $h^2$  reachability analysis, alternating between **forward** and **backward**
  - remove unreachable facts and actions



## High-Level Certificate

- ▶  $D_i$ : set of literal pairs shown dead before or in iteration  $i$
- ▶  $S_i = \{s \mid \{p, q\} \not\subseteq s \text{ for all } \{p, q\} \in D_i\}$
- ▶ start with iteration 1 and  $D_0 = \{\}$  →  $\overline{S_0} (= \{\})$  dead

### Forward iteration $i$



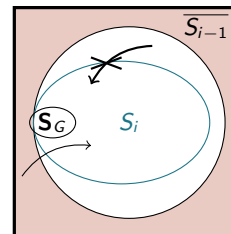
given: (1)  $\overline{S_{i-1}}$  dead

#	statement	justification
(2)	$\{!\} \subseteq S_i$	<b>B1</b>
(3)	$S_i[\mathbf{A}] \subseteq S_i \cup \overline{S_{i-1}}$	<b>B2</b>
(4)	$\overline{S_i}$ dead	<b>PI</b> with (3), (1) and (2)

## High-Level Certificate

- ▶  $D_i$ : set of literal pairs shown dead before or in iteration  $i$
- ▶  $S_i = \{s \mid \{p, q\} \not\subseteq s \text{ for all } \{p, q\} \in D_i\}$
- ▶ start with iteration 1 and  $D_0 = \{\}$  →  $\overline{S_0} (= \{\})$  dead

### Backward iteration $i$



given: (1)  $\overline{S_{i-1}}$  dead

#	statement	justification
(2)	$\overline{S_i} \cap S_G \subseteq \overline{S_{i-1}}$	<b>B1</b>
(3)	$\overline{S_i} \cap S_G$ dead	<b>SD</b> with (1) and (2)
(4)	$[A]S_i \subseteq S_i \cup \overline{S_{i-1}}$	<b>B2</b>
(5)	$\overline{S_i}$ dead	<b>RG</b> with (4), (1) and (3)

## Remarks

- ▶ representation of  $S_i$ :  $\bigwedge_{\{p,q\} \in D_i} \neg p \vee \neg q$   
 → 2CNF (Horn not suitable since  $p$  and  $q$  can be negative)
- ▶ If the  $h^2$  preprocessor detects the task unsolvable, we can extract a full proof:
  - ▶ ends in forward iteration:  $\mathbf{S}_G \subseteq \overline{S_n}$  (all goal states dead)
  - ▶ ends in backward iteration:  $\{\mathbf{I}\} \subseteq \overline{S_n}$  (initial state dead)
- ▶ Otherwise, we can use the statement “ $\overline{S_n}$  dead” to explain why we pruned certain states.
- ▶ We can also extract more fine-grained statements such as “ $S_{p \wedge q}$  dead” within the proof system.

## Recap

## Take-Home Messages

- ▶ We can verify algorithms on **different levels** (unit tests, certifying algorithms, theorem provers).
- ▶ The unsolvability proof system **incrementally** deduces knowledge about dead states.
- ▶ Its modularity enables us to **combine different sources** of information.
- ▶ Efficient verification depends on the **representation** of state sets, i.e. which operations are efficiently supported.
- ▶ Different representations can offer **tradeoffs between efficient generation and verification**.
- ▶ Generating certificates often involves **reachability** arguments.

## Future Work

- ▶ cover more planning techniques, e.g.
  - ▶ dead-end potentials
  - ▶ partial order reduction
  - ▶ task transformations
- ▶ extend the verifier
  - ▶ more representations  
 → talk in Session E2 next week about CNF
  - ▶ more inference rules
- ▶ verify the verifier!