

Certified Unsolvability in Classical Planning

2. Applications

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Certificate Structure

First look

on Github: <https://github.com/salome-eriksson/helve>

A certificate consists of the following files:

- task description
 - limited to STRIPS
 - STRIPS with negation coming soon™
 - variable and action IDs (according to occurrence in list)
- certificate
 - **state sets**: e ID type description
e 0 h p cnf 3 2 2 -1 0
 - **action sets**: a ID type description
a 0 u 2 4 5 6
 - **statements**: k ID type set-ID(s) justification
premises k 0 d 7 sd 5 4
- (BDD descriptions)

(detailed explanation in README.md of github repository)

Certificate File

Example

what we have seen so far:

#	statement	justification
(0)	\emptyset dead	ED
(1)	$\{I\} \sqsubseteq \text{states}(\neg a \vee \neg b)$	B1
(2)	$\overline{S_{\neg a \vee \neg b}}[A] \sqsubseteq S_{\neg a \vee \neg b} \cup \emptyset$	B2
(3)	$\overline{S_{\neg a \vee \neg b}}$ dead	PI with (3),(1) and (2)

...

the corresponding certificate file:

```

1  e 0 c e                7  e 3 p 2 0
2  e 1 c i                8  e 4 u 2 0
3  a 0 a                  9  k 2 s 3 4 b2
4  k 0 d 0 ed            10 e 5 n 2
5  e 2 h p cnf 2 1 1 2 0 11 k 3 d 5 pi 2 0 1
6  k 1 s 1 2 b1         ...

```

→ Demo

Blind Search

High-Level Certificate

Blind search explores all **reachable states** \mathcal{R}^Π .

→ Recall completeness proof:

Forward Blind Search Certificate

#	statement	justification
(0)	\emptyset dead	ED
(1)	$\mathcal{R}^\Pi[\mathbf{A}] \subseteq \mathcal{R}^\Pi \cup \emptyset$	B2
(2)	$\mathcal{R}^\Pi \cap \mathbf{S}_G \subseteq \emptyset$	B1
(3)	$\mathcal{R}^\Pi \cap \mathbf{S}_G$ dead	SD with (0) and (2)
(4)	\mathcal{R}^Π dead	PG with (1), (0) and (3)
(5)	$\{\mathbf{I}\} \subseteq \mathcal{R}^\Pi$	B1
(6)	$\{\mathbf{I}\}$ dead	SD with (4) and (5)
(7)	unsolvable	CI with (6)

For backwards search, show $\{\mathbf{I}\} \subseteq \overline{\mathcal{R}_B^\Pi}$ and $[\mathbf{A}]\mathcal{R}_B^\Pi \subseteq \mathcal{R}_B^\Pi \cup \emptyset$, deduce \mathcal{R}_B^Π dead (rule **RI**), and from $\mathbf{S}_G \subseteq \mathcal{R}_B^\Pi$ show \mathbf{S}_G dead.

Translation to Certificate File

#	statement	justification
(0)	\emptyset dead	ED
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead

Translation to Certificate File

#	statement	justification
(1)	$\mathcal{R}^\Pi[\mathbf{A}] \sqsubseteq \mathcal{R}^\Pi \cup \emptyset$	B2
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \sqsubseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$

Translation to Certificate File

#	statement	justification
(2)	$\mathcal{R}^\Pi \cap \mathbf{S}_G \subseteq \emptyset$	B1
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \subseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \subseteq S_0 (= \emptyset)$

Translation to Certificate File

#	statement	justification
(3)	$\mathcal{R}^\Pi \cap \mathbf{S}_G$ dead	SD with (0) and (2)
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \sqsubseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \sqsubseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead

Translation to Certificate File

#	statement	justification
(4)	\mathcal{R}^Π dead	PG with (1), (0) and (3)
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \subseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \subseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead
13	k 4 d 3 pg 1 0 3	(4) $S_3 (= \mathcal{R}^\Pi)$ dead

Translation to Certificate File

#	statement	justification
(5)	$\{\mathbf{I}\} \subseteq \mathcal{R}^\Pi$	B1
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \subseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \subseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead
13	k 4 d 3 pg 1 0 3	(4) $S_3 (= \mathcal{R}^\Pi)$ dead
14	k 5 s 1 3 b1	(5) $S_1 (= \{\mathbf{I}\}) \subseteq S_3 (= \mathcal{R}^\Pi)$

Translation to Certificate File

#	statement	justification
(6)	$\{\mathbf{I}\}$ dead	SD with (4) and (5)
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \subseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \subseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead
13	k 4 d 3 pg 1 0 3	(4) $S_3 (= \mathcal{R}^\Pi)$ dead
14	k 5 s 1 3 b1	(5) $S_1 (= \{\mathbf{I}\}) \subseteq S_3 (= \mathcal{R}^\Pi)$
15	k 6 d 1 sd 5 4	(6) $S_1 (= \{\mathbf{I}\})$ dead

Translation to Certificate File

#	statement	justification
(7)	unsolvable	CI with (6)
1	e 0 c e	$S_0 = \emptyset$
2	e 1 c i	$S_1 = \{\mathbf{I}\}$
3	e 2 c g	$S_2 = \mathbf{S}_G$
4	a 0 a	$A_0 = \mathbf{A}$
5	k 0 d 0 ed	(0) $S_0 (= \emptyset)$ dead
6	e 3 ...	$S_3 = \mathcal{R}^\Pi$
7	e 4 p 3 0	$S_4 = S_3[A_0] = \mathcal{R}^\Pi[\mathbf{A}]$
8	e 5 u 3 0	$S_5 = S_3 \cup S_0 = \mathcal{R}^\Pi \cup \emptyset$
9	k 1 s 4 5 b2	(1) $S_4 (= \mathcal{R}^\Pi[\mathbf{A}]) \sqsubseteq S_5 (= \mathcal{R}^\Pi \cup \emptyset)$
10	e 6 i 3 2	$S_6 = S_3 \cap S_2 = \mathcal{R}^\Pi \cap \mathbf{S}_G$
11	k 2 s 6 0 b1	(2) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G) \sqsubseteq S_0 (= \emptyset)$
12	k 3 d 6 sd 2 0	(3) $S_6 (= \mathcal{R}^\Pi \cap \mathbf{S}_G)$ dead
13	k 4 d 3 pg 1 0 3	(4) $S_3 (= \mathcal{R}^\Pi)$ dead
14	k 5 s 1 3 b1	(5) $S_1 (= \{\mathbf{I}\}) \sqsubseteq S_3 (= \mathcal{R}^\Pi)$
15	k 6 d 1 sd 5 4	(6) $S_1 (= \{\mathbf{I}\})$ dead
16	k 7 u ci 6	(7) unsolvable

Implementation Details

Depending on the concrete algorithm, some implementation details affect performance:

- formalism for \mathcal{R}^Π
 - symbolic search: BDD
 - overhead if no singular closed BDD!
 - explicit search: explicit enumeration or BDD
 - fast generation vs fast verification
- when to build the certificate
 - during search: unnecessary overhead for solvable problems
 - at the end: more overhead (iterate over entire closed list), but also more localized

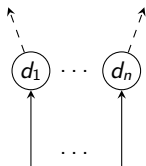
Heuristic Search

Idea

- 1 Each dead-end is dead.
- 2 The set of expanded states contains no goal state.
- 3 The set of expanded states can only reach itself and dead-ends.
- 4 → the set of expanded states is dead.
- 5 The initial state is in the set of expanded states.
- 6 → The initial state is dead and the task unsolvable.

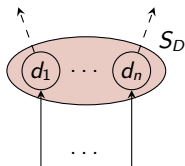
High-Level Certificate

- (1) \emptyset dead
(2) $\{d_1\}$ dead (3) $\{d_2\}$ dead ... (4) $\{d_n\}$ dead



High-Level Certificate

- (1) \emptyset dead
 (2) $\{d_1\}$ dead (3) $\{d_2\}$ dead ... (4) $\{d_n\}$ dead

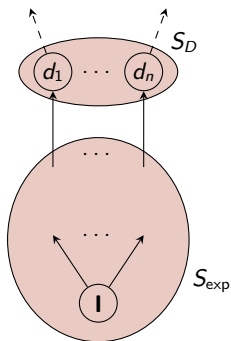


#	statement	justification
(5)	$\{d_1\} \cup \{d_2\}$ dead	UD with (2) and (3)
	\dots	
(6)	$\bigcup_{1 \leq i \leq n} d_i$ dead	UD with ...
(7)	$S_D \sqsubseteq \bigcup_{1 \leq i \leq n} d_i$	B1
(8)	S_D dead	SD with (6) and (7)



High-Level Certificate

- (1) \emptyset dead
 (2) $\{d_1\}$ dead (3) $\{d_2\}$ dead ... (4) $\{d_n\}$ dead



#	statement	justification
(5)	$\{d_1\} \cup \{d_2\}$ dead	UD with (2) and (3)
	...	
(6)	$\bigcup_{1 \leq i \leq n} d_i$ dead	UD with ...
(7)	$S_D \sqsubseteq \bigcup_{1 \leq i \leq n} d_i$	B1
(8)	S_D dead	SD with (6) and (7)
(9)	$S_{\text{exp}}[\mathbf{A}] \sqsubseteq S_{\text{exp}} \cup S_D$	B2
(10)	$S_{\text{exp}} \cap \mathbf{S}_G \sqsubseteq \emptyset$	B1
(11)	$S_{\text{exp}} \cap \mathbf{S}_G$ dead	SD with (1) and (10)
(12)	S_{exp} dead	PG with (9), (8) and (11)
(13)	$\{\mathbf{I}\} \sqsubseteq S_{\text{exp}}$	B1
(14)	$\{\mathbf{I}\}$ dead	SD with (12) and (13)
(15)	task unsolvable	CI with (14)

Bridging Representations

Statements “ $\{d_i\}$ dead” might use **different representations**.

1 Show $\{d_i\}_{\text{explicit}} \sqsubseteq \{d_i\}_{\text{R}}$ (basic statement **B4**)

and then either

2a build $(S_{\text{exp}})_{\text{explicit}}$, and

3a show $(S_{\text{exp}})_{\text{explicit}} \sqsubseteq (S_{\text{exp}})_{\text{explicit}} \cup \bigcup \{d_i\}_{\text{explicit}}$ (**B2**).

or

2b build $(S_D)_{\text{explicit}}$, $(S_D)_{\text{BDD}}$ and $(S_{\text{exp}})_{\text{BDD}}$,

3b show $(S_D)_{\text{explicit}} \sqsubseteq \bigcup \{d_i\}_{\text{explicit}}$ (**B2**),

4b show $(S_D)_{\text{BDD}} \sqsubseteq (S_D)_{\text{explicit}}$ (**B4**), and

5b show $(S_{\text{exp}})_{\text{BDD}} \sqsubseteq (S_{\text{exp}})_{\text{BDD}} \cup (S_D)_{\text{BDD}}$ (**B2**).

→ tradeoff efficient generation vs efficient verification

Delete Relaxation

h^{\max} dead-end

$h^{\max}(s) = \infty \leftrightarrow$ some $g \in \mathbf{G}$ relaxed unreachable

Consider $R_u^+(s) = \{v \mid v \text{ relaxed unreachable from } s\}$ and

$$\varphi = \bigwedge_{v \in R_u^+(s)} \neg v.$$

- We can't reach any s' containing any $v \in R_u^+(s)$: $S_\varphi[\mathbf{A}] \subseteq S_\varphi$
- All states satisfying φ do not satisfy g : $S_\varphi \cap \mathbf{S}_G = \emptyset$
- State s satisfies φ : $\{s\} \subseteq S_\varphi$

→ Show that S_φ is dead (**PG**) and thus s is dead (**SD**).

We can choose between different **representations**:

BDD, Horn formula, 2CNF formula, explicit (over $R_u^+(s)$)

h^m & Clause-Learning State Space Search

h^m dead-ends:

- same concept as h^{\max} : $\varphi = \bigwedge_{t \in R_u^m(s)} \bigvee_{v \in t} \neg v$, where $R_u^m(s)$ are the tuples unreachable from s .
- representation: Horn formulas (or 2CNF formulas for $m = 2$)
→ **BDDs not suited** [Edelkamp & Kissmann 2011]

Clause-Learning State Space Search [Steinmetz & Hoffmann (2017)]:

- uses h^C → same concept (again)
- can be refined to detect **I** as dead-end → compact certificate
- uses additional source for mutexes
→ **Integrate additional information into certificate!**

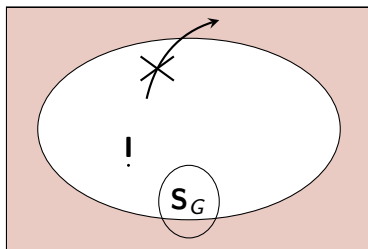
Other heuristics

- h^{\max} approach covers **all** delete-relaxation heuristics ($h^{\text{LM-Cut}}$, landmarks based on delete relaxation, ...)
- Merge & Shrink:
 - transformation from Merge & Shrink representation to ADD [Helmert et al. 2014] and extract ∞ -paths to BDD
→ limited to **linear merge strategies** [Helmert et al. 2015]
 - one set for **all dead-ends**
→ certificate more compact
 - implementation detail: unreachable and dead-end states merged
→ **disable for certificate generation**

h^2 Preprocessor

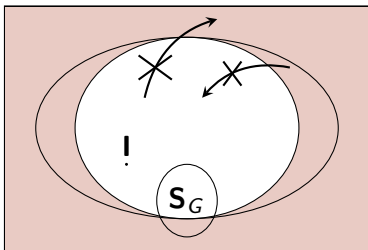
Algorithm Overview

- introduced in [Alcázar & Torralba (2015)]
- preprocessing step that simplifies planning task
- used in many IPC planners
- **incremental** h^2 reachability analysis, alternating between **forward** and **backward**
 - remove unreachable facts and actions



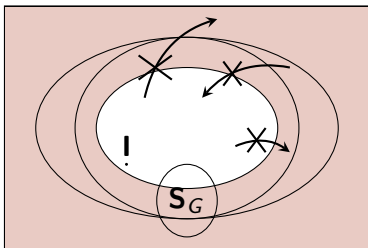
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Algorithm Overview

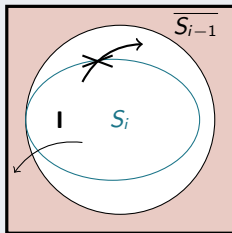
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- preprocessing step that simplifies planning task
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- **incremental** h^2 reachability analysis, alternating between **forward** and **backward**
 - remove unreachable facts and actions



High-Level Certificate

- D_i : set of literal pairs shown dead before or in iteration i
- $S_i = \{s \mid \{p, q\} \not\subseteq s \text{ for all } \{p, q\} \in D_i\}$
- start with iteration 1 and $D_0 = \{\}$ $\rightarrow \overline{S_0} (= \{\})$ dead

Forward iteration i



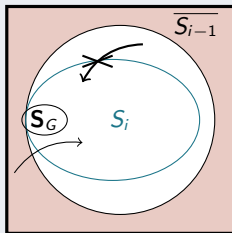
given: (1) $\overline{S_{i-1}}$ dead

#	statement	justification
(2)	$\{I\} \subseteq S_i$	B1
(3)	$S_i[A] \subseteq S_i \cup \overline{S_{i-1}}$	B2
(4)	$\overline{S_i}$ dead	PI with (3), (1) and (2)

High-Level Certificate

- D_i : set of literal pairs shown dead before or in iteration i
- $S_i = \{s \mid \{p, q\} \not\subseteq s \text{ for all } \{p, q\} \in D_i\}$
- start with iteration 1 and $D_0 = \{\}$ $\rightarrow \overline{S_0} (= \{\})$ dead

Backward iteration i



given: (1) $\overline{S_{i-1}}$ dead

#	statement	justification
(2)	$\overline{S_i} \cap \mathbf{S}_G \subseteq \overline{S_{i-1}}$	B1
(3)	$\overline{S_i} \cap \mathbf{S}_G$ dead	SD with (1) and (2)
(4)	[A] $S_i \subseteq S_i \cup \overline{S_{i-1}}$	B2
(5)	$\overline{S_i}$ dead	RG with (4), (1) and (3)

Remarks

- representation of S_i : $\bigwedge_{\{p,q\} \in D_i} \neg p \vee \neg q$
→ 2CNF (Horn not suitable since p and q can be negative)
- If the h^2 preprocessor detects the task unsolvable, we can extract a full proof:
 - ends in forward iteration: $\mathbf{S}_G \subseteq \overline{S_n}$ (all goal states dead)
 - ends in backward iteration: $\{\mathbf{I}\} \subseteq \overline{S_n}$ (initial state dead)
- Otherwise, we can use the statement “ $\overline{S_n}$ dead” to explain why we pruned certain states.
- We can also extract more fine-grained statements such as “ $S_{p \wedge q}$ dead” within the proof system.

Recap

Take-Home Messages

- We can verify algorithms on **different levels** (unit tests, certifying algorithms, theorem provers).
- The unsolvability proof system **incrementally** deduces knowledge about dead states.
- Its modularity enables us to **combine different sources** of information.
- Efficient verification depends on the **representation** of state sets, i.e. which operations are efficiently supported.
- Different representations can offer **tradeoffs between efficient generation and verification**.
- Generating certificates often involves **reachability** arguments.

Future Work

- cover more planning techniques, e.g.
 - dead-end potentials
 - partial order reduction
 - task transformations
- extend the verifier
 - more representations
 - talk in Session E2 next week about CNF
 - more inference rules
- verify the verifier!