Epistemic Planning: Semantic Approach

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http://www2.compute.dtu.dk/~tobo/children_cabinet_cropped.mp4
Epistemic planning =
automated planning + Theory of Mind reasoning

**Aim:** To compute plans that can take the mental states of other agents into account.

**Essentially:** (Decentralised) **multi-agent planning** in environments with (potentially higher-order) **information asymmetry**.

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Automated planning

Logical reasoning about the mental states of other agents
Syntactic vs semantic, explicit vs implicit

When moving from standard propositional states to states including a Theory of Mind, there are two distinct paths one might take.

- **Syntactic approach:** States are (sets of) formulas (e.g. formulas of S5 epistemic logic)
- **Semantic approach:** States are semantic models (e.g. epistemic models = Kripke models).

Note: For propositional planning under full observability, the approaches are trivially equivalent.

Furthermore, for the semantic approach, there is a choice between:

- **Explicit approach:** Full state space is assumed given, and solution concept is defined directly in terms of this. E.g. logics like ATEL and CSL. [van der Hoek and Wooldridge, 2002, Jamroga and Aagotnes, 2007]
- **Implicit approach:** State space is induced by initial state and action library (as in classical STRIPS/PDDL planning).

DEL-based epistemic planning is *implicit* and *semantic*. [Bolander and Andersen, 2011]
Epistemic states: Multi-pointed epistemic models of multi-agent S5. Nodes are worlds. Designated worlds: ○ (those considered possible by planning agent).
The coordinated attack problem in dynamic epistemic logic (DEL)

Two generals (agents), $a$ and $b$. They want to coordinate an attack, and only win if they attack simultaneously.

d: “general $a$ will attack at dawn”.

$m_i$: the messenger is at general $i$ (for $i = a, b$).

Initial epistemic state:

\[ s_0 = \begin{array}{c}
\text{worlds} \\
\text{edges}
\end{array} \]

Nodes are **worlds**, edges are **indistinguishability edges** (reflexive loops not shown).
The coordinated attack problem in dynamic epistemic logic (DEL)

Recall: $d$ means “$a$ attacks at dawn”; $m_i$ means messenger is at general $i$.

Available epistemic actions (aka action models aka event models):

$$a:send = \begin{array}{c}
\text{pre: } d \land m_a \\
\text{post: } m_b \land \neg m_a \\
\end{array}$$

And symmetrically an epistemic action $b:send$. We read $i:\alpha$ as “agent $i$ does $\alpha$”.

Nodes are events, and each event has a precondition and a postcondition (effect). The precondition is an epistemic formula and the postcondition is a conjunction of literals.

[Baltag et al., 1998, van Ditmarsch and Kooi, 2008]
The product update in dynamic epistemic logic

\[
\begin{align*}
{s_0} &= \bigl( d, m_a \bigr) \quad b \quad \bigl( m_a \bigr) \\
\begin{array}{ll}
{w_1^0} & \quad {w_2^0}
\end{array} \\
\end{align*}
\]

\[
\begin{align*}
a:send &= \quad \frac{\text{pre: } d \land m_a}{\text{post: } m_b \land \neg m_a} \\
\begin{array}{ll}
e_1 & \quad a \quad e_2
\end{array} \\
\end{align*}
\]

\[
\begin{align*}
{s_0} \otimes a:send &= \bigl( d, m_b \bigr) \quad a \quad \bigl( d \bigr) \\
\begin{array}{ll}
{w_1^1} & \quad {w_2^1} & \quad {w_3^1}
\end{array} \\
\end{align*}
\]

\[
\begin{align*}
{s_0} \otimes a:send &\models K_a d \land K_b d \land \neg K_a K_b d
\end{align*}
\]
\[ s_0 = w_0 \]

\[ s_1 = s_0 \otimes a:send = w_1 \]

\[ s_2 = s_1 \otimes b:send = w_2 \]

\[ s_3 = w_3 \]
**Epistemic planning tasks**

**Definition.** An *epistemic planning task* (or simply a *planning task*) $T = (s_0, A, \gamma)$ consists of an epistemic state $s_0$ called the *initial state*; a finite set of epistemic actions $A$; and a *goal formula* $\gamma$ of the epistemic language.

**Definition.** A (sequential) *solution* to a planning task $T = (s_0, A, \gamma)$ is a sequence of actions $\alpha_1, \alpha_2, \ldots, \alpha_n$ from $A$ such that for all $1 \leq i \leq n$, $\alpha_i$ is applicable in $s_0 \otimes \alpha_1 \otimes \cdots \otimes \alpha_{i-1}$ and

$$s_0 \otimes \alpha_1 \otimes \alpha_2 \otimes \cdots \otimes \alpha_n \models \gamma.$$

**Example.** Let $s_0$ be the initial state of the coordinated attack problem. Let $A = \{a:send, b:send\}$. Then the following are planning tasks:

1. $T = (s_0, A, Cd)$, where $C$ denotes common knowledge. It has no solution.

2. $T = (s_0, A, E^n d)$, where $E$ denotes “everybody knows” and $n \geq 1$. It has a solution of length $n$.

[Bolander et al., 2020]
Epistemic planning example: Get the cube

- **Objects**: $\mathcal{O} = \{b_1, b_2, c\}$, two boxes $b_1$ and $b_2$, and a cube $c$.
- **Agents**: $\mathcal{A} = \{h, a\}$, a human $h$ and a robot $r$. The robot is the planning agent.
- **Atomic propositions**: $\text{In}(x, y)$ means $x$ is in $y$, where $x, y \in \mathcal{O} \cup \mathcal{A}$ (when $y \in \mathcal{A}$, it means $y$ is holding $x$).

Initial epistemic state:

$$s_0 = \begin{array}{c}
\text{In}(c, b_1) \\
\text{In}(c, b_2)
\end{array}$$

The goal is for the human to hold the red cube, $\text{In}(r, h)$. 
Actions specialised for the case of $\mathcal{O} = \{b_1, b_2, c\}$.

Agent $i$ (semi-privately) **peeks** into box $x$:

\[
i:\text{peek}(x) = \begin{array}{l}
\text{pre: } ln(c, x) \\
\text{post: } \neg ln(c, x)
\end{array}
\]

Agent $i$ (publicly) **picks up** object $x$ from $y$:

\[
i:\text{pickup}(x, y) = \begin{array}{l}
\text{pre: } ln(x, y) \\
\text{post: } ln(x, i) \land \neg ln(x, y)
\end{array}
\]

Agent $i$ (publicly) **puts** object $x$ in $y$:

\[
i:\text{putdown}(x, y) = \begin{array}{l}
\text{pre: } ln(x, i) \\
\text{post: } ln(x, y) \land \neg ln(x, i)
\end{array}
\]

Agent $i$ (publicly) **announces** that formula $\varphi$ is true:

\[
i:\text{ann}(\varphi) = \begin{array}{l}
\text{pre: } \varphi
\end{array}
\]
Get the cube: Planning task and solutions

The planning task $T$ has the actions of the previous slide and initial state $s_0$ and goal $\gamma$ given by:

$$ s_0 = \text{In}(c, b_1) \quad h \quad \text{In}(c, b_2) $$

$$ \gamma = \text{In}(r, h) $$

Solution to $T$, by robot $R$:

$$ s_0 = \text{In}(c, b_1) \quad h \quad \text{In}(c, b_2) $$

$$ s_1 = s_0 \otimes r:\text{pickup}(c, b_1) = \text{In}(c, r) $$

$$ s_2 = s_1 \otimes r:\text{putdown}(c, h) = \text{In}(c, h) $$
Applicability, perspective shifts, implicit coordination

Seemingly simpler solution: \( h: \text{pickup}(c, b_1) \). But intuitively, this shouldn’t work, since the human doesn’t know the cube is in box 1...

**Applicability:** An action \( \alpha \) is **applicable** in a state \( s \) if for each designated world \( w \) of \( s \) there is a designated event \( e \) of \( \alpha \) with \( w \models \text{pre}(e) \).

**Perspective shift:** The **perspective shift** of state \( s \) to agent \( i \), denoted \( s^i \), is achieved by closing under the indistinguishability relation of \( i \). We call \( s^i \) the **perspective** of agent \( i \) on state \( s \).

\[
\begin{align*}
s_0 &= \text{In}(c, b_1) \xrightarrow{h} \text{In}(c, b_2) \\

s_h &= \text{In}(c, b_1) \xrightarrow{h} \text{In}(c, b_2)
\end{align*}
\]

**Example.** \( h: \text{pickup}(c, b_1) \) is not applicable in \( s_0 \) from \( h \)'s perspective.

**Implicitly coordinated solution to planning task:** Each action has to be applicable from the perspective of the acting agent; and the product update \( s \otimes i: \alpha \) is replaced by \( s^i \otimes i: \alpha \).
Get the cube: Implicit coordination

Joint solution to $T$, by robot $R$, implicitly coordinated:

$$s_0 = \text{In}(c, b_1)$$

$$s_1 = s_0 \otimes r:\text{ann}(\text{In}(c, b_1)) = \text{In}(c, b_1)$$

$$s_2 = s_1 \otimes h:\text{pickup}(c, b_1) = \text{In}(c, h)$$

If purely epistemic actions (announcements) have a lower cost than ontic actions (moving things around), the solution above is the only optimal one.
Undecidability: lengthening and shortening chains

Consider a chain produced by the coordinated attack problem:

Using preconditions of modal depth 1 we can shorten the chain by 1:

We can now both lengthen (by send) and shorten chains (by shorten), and this allows us to encode two-counter machines \(\Rightarrow\) undecidability of the plan existence problem!

Undecidability holds even with preconditions of modal depth 1, and for purely epistemic planning (no postconditions) even for modal depth 2.  
[Bolander and Andersen, 2011, Charrier et al., 2016, Bolander et al., 2020]
Some of the current challenges in epistemic planning

- **Undecidability issues**: open complexity problems.
  [Bolander et al., 2020]

- **State size explosion problems**: find compact state representations.
  [Charrier and Schwarzentruber, 2017, van Benthem et al., 2018]

- **The belief-revision problem in DEL**: How to recover from false beliefs without an underlying epistemic relation. Relates to the state size explosion problem.

- **Heuristics for epistemic planning**: to reduce all of the above mentioned complexity and scalability issues

- **Languages**: syntactic languages for describing actions.
  [Baral et al., 2012, Baral et al., 2013]

This, and much more, is discussed in the “Epistemic Planning” special issue of AIJ currently being finalised.
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