ICAPS2020 Summer School: Temporal Planning

Arthur $\mathsf{Bit}\text{-}\mathsf{Monnot}^1$ and $\mathsf{Andrea}\ \mathsf{Micheli}^2$ 14 October 2020

¹LAAS-CNRS, Toulouse, France. abitmonnot@laas.fr ²Fondazione Bruno Kessler, Trento, Italy. amicheli@fbk.eu

Lecture program

- 1. Introduction: what is temporal planning?
 - Motivation
 - Applications
 - Complexity
- 2. Temporal reasoning
 - Temporal Networks Generalities
 - Point Algebra & Allen Algebra
 - Simple Temporal Networks
 - Advanced temporal reasoning frameworks
- 3. State-oriented techniques
 - Languages & Complexity
 - Time in forward search
 - State-based SAT/SMT/MIP encoding
- 4. Time-oriented techniques
 - Building blocks for action/timeline representation
 - Search Process
 - Planning as Scheduling
- 5. Conclusion

Introduction to temporal planning

Given a model of a system and a goal to be reached, find a course of actions to drive the system to the goal.



Initial Configuration and Goal

Given a model of a system and a goal to be reached, find a course of actions to drive the system to the goal.



Al Planning is a purely deductive technique: all the information and all the solutions are in the input, the planner reasons on the model and extracts them.

Example



(taken from "Automated Planning and Acting" by M. Ghallab, D. Nau and P. Traverso)

• Time and temporal constraints must be considered

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Example

Possible Plan:

- A: move(r1, d1, d3) at 0 with duration 7
- B: *load*(*r*2, *c*3, *p*2) at 8 with duration 9



- Time and temporal constraints must be considered
- Actions can overlap and interfere

Example

Possible Plan:

- A: *move*(*r*1, *d*1, *d*3) at 0 with duration 7
- B: *load*(*r*2, *c*3, *p*2) at 6 with duration 9



- Time and temporal constraints must be considered
- Actions can overlap and interfere
- The key intuition is that we want to decide both what to do and when to do it: we want to generate a Gantt-chart of activities

Example

Possible Plan:

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Why Temporal Planning?



- Optimal control in known (or partially-known) environments (e.g. Factories, Warehouses)
- Mission planning (e.g. Exploration rovers, UAV, AUV)
- Safety critical control and recovery (e.g. Satellites)

Time is a first-class citizen and can be interpreted on different domains

- Discrete-time
 - Events happen at discrete steps (e.g. \mathbb{Z})
 - We do not model what happens between two steps
 - A planner has to find a way to schedule events in the discrete steps
 - Q: how many (non-overlapping) actions with duration > 0 can you fit in 10 time-units?
- Dense-time
 - Events can happen in a dense continuum (e.g. ${\mathbb Q}$ or ${\mathbb R})$
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Warning

Discrete-time seems simpler (and sometimes is), but consider that linear programming over the rationals is polynomial (ellipsoid algorithm) while over the integers it is NP-complete (integer programming)!

Theorem ([Rintanen, 2007, Gigante et al., 2020a, Gigante et al., 2017]) *Temporal planning with discrete time is EXPSPACE-Complete.*

Theorem ([Gigante et al., 2020a, Bozzelli et al., 2020]) *Temporal planning with dense time is undecidable.*

Temporal Reasoning

Why Temporal Reasoning?



Answering questions about a (partial?) plan:

- Consistency (no deadlocks, meet deadlines, ...)
- Occurrence times (earliest/latest start times, ...)
- Relative ordering (action A before action B?)
- Dispatchability (strategy to execute the plan)

Interval

A contiguous subset of time, characterized by a start time s and a duration d.

Timepoint

A single point in time, typically related to a particular instantaneous event.





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Temporal Networks

Graphical models where:

- nodes = temporal references
 - timepoints, intervals
- edges = temporal relations
 - before, after, ...



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Reasoning rules: Composition

 n_1 r_{12} n_2 r_{23} n_3 $r_{12} \circ r_{23}$

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Reasoning rules:

Composition





Ordering

Given 2 timepoints *a* and *b*, exactly one of the following will be true:

- *a* < *b* (*a* before *b*)
- a > b (a after b)
- a = b (same occurrence time)

Point Algebra: Qualitative ordering between timepoints

Relations:

Ordering

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- a > b (a after b)
- a = b (same occurrence time)

Relation

A relation $a \xrightarrow{R} b$ restricts the set R of possible ordering between a and b.

R	Meaning		
$\{<\}$	before		
$\{>\}$	after		
$\{=\}$	equal		
$\{<,=\}$	before or equal (\leq)		
$\{>,=\}$	after or equal (\geq)		
$\{<,>\}$	different ($ eq$)		
{}	contradiction		
$\{<,=,>\}$	tautology (\top)		

Examples: $a \xrightarrow{\{<\}} b$ a must be before b $a \xrightarrow{\{<,=\}} b$ a must be before or equal to b

Given
$$A \xrightarrow{r_{AB}} B \xrightarrow{r_{BC}} C$$

- infer $A \xrightarrow{r_{AC}} C$
- where $r_{AC} = r_{AB} \circ r_{BC}$

Composition table:





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Composition table:





- Given $A \xrightarrow{r_1} B$ and $A \xrightarrow{r_2} B$
 - infer $A \xrightarrow{r} B$
 - where $r = r_1 \cap r_2$
- Property: $A \xrightarrow{r_1} B$ and $A \xrightarrow{r_2} B$
 - are weaker than $A \xrightarrow{r_1 \cap r_2} B$
 - become redundant and can be removed



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For each nodes i, j, k:

- $r_{ik} \leftarrow r_{ik} \cap (r_{ij} \circ r_{jk})$
- repeat to quiescence



- Infers all pair-wise orderings
- Network is consistent if all pairs of edges have a valid ordering (i.e. no $a \stackrel{\{\}}{\longrightarrow} b$)

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Qualitative orderings between intervals



Allen's Interval Algebra: Relations

Relation	i j	Relation	i j
	i		i
before (b)	j	start (s)	j
	i		i
meet (m)	j	during (d)	j
	i		i
overlap (o)	j	finish (f)	j



+ all 6 inverse: i before j \Leftrightarrow j before' i
- Intersection operation (set intersection)
- Composition matrix

•	b	m	0	8	d	f	b'	d'	f'
b	b	b	b	b	$u \cup v$	$u \cup v$	P	b	b
m	b	b	b	m	v	v	$u' \cup v'$	b	b
0	b	b	u	0	v	v	$u' \cup v'$	$u \cup w'$	u
- 8	b	b	u	8	d	d	b'	$u \cup w'$	u
d	b	b	$u \cup v$	d	d	d	b'	P	$u \cup v$
f	m	m	v	d	d	f	b'	$u' \cup v'$	x
b'	P	$w \cup u'$	$w \cup u'$	$w \cup u'$	$w \cup u'$	b'	b'	b'	b'
d'	$u \cup w'$	v'	$o' \cup w'$	$o \cup w'$	y	v'	$u' \cup v'$	d'	d'
f'	b	m	0	0	v	x	$u' \cup v'$	d'	f'
s'	$u \cup w'$	$o \cup w'$	$o \cup w'$	$\{s, e, s'\}$	$o' \cup w$	0'	b'	d'	d'



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m	b	b	b	m	v	v	$u' \cup v'$	b	b
0	b	b	u	0	v	v	$u' \cup v'$	$u \cup w'$	u
8	b	b	u	8	d	d	b'	$u \cup w'$	u
d	b	b	$u \cup v$	d	d	d	b'	P	$u \cup v$
f	m	m	v	d	d	f	b'	$u' \cup v'$	x
b'	P	$w \cup u'$	$w \cup u'$	$w \cup u'$	$w \cup u'$	b'	b'	b'	b'
d'	$u \cup w'$	v'	$o' \cup w'$	$o \cup w'$	y	v'	$u' \cup v'$	d'	d'
f'	b	m	0	0	v	x	$u' \cup v'$	d'	f'
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0	b	b	u	0	v	v	$u' \cup v'$	$u \cup w'$	u
8	b	b	u	8	d	d	b'	$u \cup w'$	u
d	b	b	$u \cup v$	d	d	d	b'	P	$u \cup v$
f	m	m	v	d	d	f	b'	$u' \cup v'$	x
b'	P	$w \cup u'$	$w \cup u'$	$w \cup u'$	$w \cup u'$	b'	b'	b'	b'
d'	$u \cup w'$	v'	$o' \cup w'$	$o \cup w'$	y	v'	$u' \cup v'$	d'	d'
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8	b	b	u	8	d	d	b'	$u \cup w'$	u
d	b	b	$u \cup v$	d	d	d	b'	P	$u \cup v$
f	m	m	v	d	d	f	b'	$u' \cup v'$	x
b'	P	$w \cup u'$	$w \cup u'$	$w \cup u'$	$w \cup u'$	b'	b'	b'	b'
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8	b	b	u	8	d	d	b'	$u \cup w'$	u
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b'	P	$w \cup u'$	$w \cup u'$	$w \cup u'$	$w \cup u'$	b'	b'	b'	b'
d'	$u \cup w'$	v'	$o' \cup w'$	$o \cup w'$	y	v'	$u' \cup v'$	d'	d'
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Can be expressed into Point Algebra by relations on start/end timepoints

Simple Temporal Network (STN)

Quantitative ordering between timepoints

Allowed to bound the delay between two timepoints:

$-10 \le t_b - t_a \le 10$	t_a and t_b should not occur	$a \xrightarrow{[-10,10]} b$
	more than 10s appart	
$5 \leq t_b - t_a \leq 7$	t_b must occur between 5 and 7	$a \xrightarrow{[5,7]} b$
	seconds after t_a	

Also known as difference logic in automated reasoning community

STN: Composition & Intersection



STN: Composition & Intersection



STN: Distance Graph

Equivalent

$$1 \leq t_b - t_a \leq 2 \qquad \qquad \boxed{t_a} \qquad \boxed{[1,2]} \qquad \qquad \boxed{t_b} \\ -2 \leq t_a - t_b \leq -1 \qquad \qquad \boxed{t_a} \qquad \boxed{[-2,-1]} \qquad \boxed{t_b}$$

STN: Distance Graph



Distance graph: only allow upper bounds on difference:







Composition: $(a \xrightarrow{2} b) \circ (b \xrightarrow{10} c) = a \xrightarrow{2+10} c$ Intersection: $(a \xrightarrow{1} b) \circ (a \xrightarrow{5} b) = a \xrightarrow{\min(1,5)} b$



- Length of shortest path from X to Y:
 - \bullet = max delay from X to Y
 - \bullet = -min delay from Y to X



- Length of shortest path from X to Y:
 - $\bullet \ = \max \ \text{delay from X to Y}$
 - $\bullet = \mathsf{min} \mathsf{ delay from Y to X}$

Why?

- Path is a composition (sum) of edges
- Shortest path is the one with minimum length (intersection = min)

STN: Earliest/Latest start time of timepoints

Bellman-Ford Algorithm:

- One-to-All shortest path
 - forward graph: max delay
 - backward graph: min delay
- From ORIGIN timepoint
- Complexity: $O(|N| \times |E|)$
- Incremental: [Cesta and Oddi, 1996]

Possible delay from *O* to ...:

а	[1,2]
b	[6, 12]
С	[3,5]



Floyd-Warshall Algorithm:

- All-Pairs shortest path
- Complexity: $O(|N|^3)$
- Incremental: [Planken, 2008]

-u	ax acity fromto								
		0	а	b	С				
	0	0	2	12	5				
	а	-1	0	10	3				
	b	-6	-5	0	-2				
	с	-3	-2	8	0				

Max delay from ... to ...:



An STN is **consistent** if and only if there is no cycle of negative length in the distance graph.

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Deadlock: A before B & B before A:



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Deadlock: A before B & B before A:





Edges of a negative cycle constitute an unsatisfiable subset of constraints.

- To make the STN consistent, at least one of those constraints must be relaxed
- Can be provided by classical algorithms (Bellman-Ford, ...)
- Exploited by some planners [Rankooh and Ghassem-Sani, 2015]

 $\mathsf{STN} \text{ extensions:}$

- STN with Uncertainty [Vidal and Fargier, 1999]
- Disjunctive STN [Tsamardinos and Pollack, 2003]
- Conditional STN [Combi et al., 2013]
- Multi-Agent STN [Casanova et al., 2015]
- Time-Dependent STN [Pralet and Verfaillie, 2012]

Execution of an STN (dispatching): [Tsamardinos and Pollack, 2003]

State-oriented techniques

Action-based languages

PDDL: [Fox and Long, 2003]

```
(define (domain moves)
 2
     (:requirements :strips :typing)
 3
     (:types dock robot)
 4
     (:predicates
 5
        (loc ?r - robot ?d - dock)
 6
        (occupied ?x - dock))
 7
 8
     (:durative-action move
 9
        :parameters(
10
         ?r - robot
11
          ?src ?dst - dock)
12
        :duration (and (>= ?duration 5)
13
                        (\langle = ?duration 10 \rangle)
14
        :precondition
15
        (and (at start (not (occupied ?dst)))
16
                (at start (loc ?r ?src)))
17
        effect
18
          (and (at end (not (occupied ?src)))
19
               (at start (occupied ?dst))
20
                (at start (not (loc ?r ?src)))
21
                (at end (loc ?r ?dst)))))
22
23
    (define (problem p) (:domain moves)
24
     (:objects r1 r2 - robot
25
               d1 d2 d3 - dock
26
     (:init (occupied d1) (occupied d2)
27
            (loc r1 d1) (loc r2 d2))
28
     (:goal (loc r1 d2)))
```

```
ANML [Smith et al., 2008]
 1 type Dock;
 3 fluent boolean occupied(Dock d);
 5 type Robot with {
 6
    fluent Dock loc:
 8
     action move(Dock dst) {
 9
       duration \geq 5 and duration \leq 10:
10
       [start] not occupied(dst);
       [end] loc := dst:
11
12
       [end] occupied(loc) := false;
13
       [end] occupied(dst) := true:
14 };
15 };
16
17 instance Robot r1. r2:
18 instance Dock d1, d2, d3:
19
20 [start] occupied(d1) := true:
21 [start] occupied(d2) := true;
22 [start] r1.loc := d1:
23 [start] r2.loc := d2:
24
25 [end] r1.loc == d2;
```

Point-based languages: NDL [Rintanen, 2015b] and TPP [Micheli and Scala, 2019]. 27/61

Some languages (most notably PDDL 2.1 and PDDL+) require " ϵ -separation"

- When two events interfere (e.g. they cannot be freely reordered) they need to be separated by a minimum quantum of time ϵ
 - For example, consider the event of taking the phone and the event of placing a call: one must causally happen before the other
- If ϵ is fixed a-priori, the time interpretation is effectively discrete [Gigante et al., 2020a]
- If the planner needs to find an ϵ that works then time interpretation might be dense

Required concurrency [Cushing et al., 2007]

Definition

A plan is concurrent when there exists a time t at which more than one action is running, otherwise the plan is sequential. A solvable planning problem has required concurrency when all solutions are concurrent.

Matchcellar

You have some matches and a number of fuses to mend in a dark room. You can only mend a fuse while a match is lit.



Computational Complexity without Action Self-Overlapping

Theorem ([Rintanen, 2007, Gigante et al., 2020a])

Action-based temporal planning without action self-overlapping is PSPACE-Complete.

Action self-overlapping

A plan contains action self-overlapping if there exists a time t at which two or more instances of the same ground action are running:



Action self-overlapping is rarely useful in practice [Fox and Long, 2007].

- 1. Find a plan π for the classical planning problem having one instantaneous action for each durative action with the same (pre)conditions and effects
- 2. Post-process the plan, attaching durations to actions and de-ordering [Bäckström, 1998] the events to reduce the makespan.

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Example

```
Time-abstracted plan: A;B
(Suppose start(A) supports start(B))
```

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Drawback: incomplete for problems with required concurrency; non-makespan optimal for temporally-simple

Basic idea: split each durative action A into two instantaneous actions: start(A) and end(A)



The search method needs to ensure that for each start(A) there exists exactly one end(A) and that durative conditions are respected.

The problem with time and heuristic search



Splitting over (dense) time!

The problem with time and heuristic search



Splitting over (dense) time!

Decision Epoch Planning

Idea

- Augment the planning state with an agenda of open actions and a timestamp *t*
- At each step, we can either start a new action instance or advance time
- Time can advance only to the first event in the agenda: the agenda is then updated to reflect this time pass
- A goal state must satisfy the goal conditions and have an empty agenda

Suppose dur(A) = 5 and dur(B) = 2



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Several planners use decision epoch: SAPA [Do and Kambhampati, 2003], TLPlan [Bacchus and Ady, 2001], TFD [Eyerich et al., 2012], ...
Decision Epoch is still incomplete [Cushing et al., 2007]

Decision Epoch planners cannot start actions at arbitrary times: only at the start or end of another action in the agenda



Idea

- Augment the planning state with an agenda of future events and an STN of past and future time-points
- Time advance is implicit: a path in the search tree represents a total (sometimes a partial) order of the events
- A goal state must satisfy the goal conditions, have an empty agenda and have a consistent STN



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- A goal state must satisfy the goal conditions, have an empty agenda and have a consistent STN



Idea

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Suppose dur(A) = 5 and dur(B) = 2



A family of planners use this approach: Crikey [Coles et al., 2009], POPF [Coles et al., 2010], COLIN [Coles et al., 2012], OPTIC [Benton et al., 2012]. TAMER [Valentini et al., 2020] also uses this idea for ANML. One important note: in planning we are used to graph search spaces, but how can we recognize that two states with temporal information are equal?

- In Decision-epoch is simple: two states are equal if the have the same fluent assignment, the same agenda and the same timestamp
- In forward heuristic search with symbolic time it is hard: we need to understand if two STNs are isomorphic to decide if two states are equal
 - We can under-approximate equality [Coles and Coles, 2016]

- Classical planning relaxations [Vidal, 2011, Valentini et al., 2020]
 - Relax the temporal aspects of the planning domain and use h^+ , h^{add} , ...
- Context-enhanced Additive Heuristic for Temporal Planning [Eyerich et al., 2012]
 - Transform decision-epoch times into action costs and use h^{cea} [Helmert and Geffner, 2008]
- Temporal Relaxed Planning Graph [Coles et al., 2010, Coles and Coles, 2017]
 - Compute approximated the plan length using TRPG generated by computing minimal time of fact and action layers

As in SATPlan [Kautz and Selman, 1992]: encode the bounded planning problem as a formula

- Any model of the formula encodes a valid plan
- If the formula is unsatisfiable, either the problem is unsolvable or the bound is insufficient and needs to be increased

In order to encode time, SAT is not enough, we need numeric quantities and linear arithmetic: $SMT(\mathcal{LA})$

Satisfiability Modulo Theory (SMT) [Barrett et al., 2009] is the problem of deciding the satisfiability of a first-order formula expressed in a given (decidable) theory T.

SMT at-a-glance

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Example

$$\phi \doteq (x > 2) \land (x < 8) \land ((x < 1) \lor (x > 7))$$

- Is satisfiable in the theory of linear rational arithmetic because $\{x \doteq 7.5\} \models \phi$
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Practical features

- Many theories are supported
- Efficient and well-supported implementations
- Common language to express problems (SMT-LIB)
- Annual solver competitions since 2003

- Fluent values are represented at each step by SMT variables: v^i for each $i \in 1, \cdots, k$
 - A step corresponds to one time point in the timeline

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- Action durations are enforced with $\mathcal{LRA}/\mathcal{LIA}$

• e.g.
$$\bigwedge_{i=0}^k s^i_a \rightarrow \bigvee_{j=i+1}^k (e^j_a \wedge t^j - t^i = dur(a))$$

Each encoding is different and has strengths and weaknesses, here is a general gist:

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 $\bullet\,$ A "frame-axiom" enforces that values can only change when there is an effect

• e.g.
$$\bigwedge_{i=0}^{k-1} v^i \neq v^{i+1} \rightarrow a^i$$

- TM-LPSAT [Shin and Davis, 2005] planning with processes and continuous change using SMT
- Planning with clock variables in SMT [Rintanen, 2015a]
- LCP [Bit-Monnot, 2018], an SMT-based domain independent planner for ANML
- MIP-based temporal planning [Dimopoulos and Gerevini, 2002]

Advanced: Intermediate Condition and Effects

Intermediate conditions and effects (ICEs)

Actions with conditions and effects defined at arbitrary moments during the action's duration.



Supported by ANML: LCP [Bit-Monnot, 2018] and TAMER [Valentini et al., 2020].

Time-oriented techniques

Key questions in (temporal) planning:

What actions?Tied by grounding into operatorsWhich parameters?Fixed by progression search

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In state-oriented forward search, all 3 decisions made at once:

- necessary for extracting a state
- error prone (high branching factor)
- enables extremely helpful search heuristics

Plans & Partial Orders





Plans & Partial Orders



Plans & Partial Orders



Timelines

Timeline

A timeline denotes the evolution of a particular state variable through time.



Token

A timeline is filled with tokens, that give the value of the state variable over an interval of time.

$$\boxed{ \text{loc}(\text{Alice}) = \text{A} }_{t_s} t_{e}$$

Action-based model:

When some conditions hold, then you can produce this effect



Planners: IxTeT, FAPE, CPT, LCP

Timeline-based model:

The presence of a token implies additional conditions.



Action-based models can be expressed as timeline-based models [Gigante et al., 2016]

Effect tokens

Value v of a state variable sv over a temporal interval $[t_s, t_e]$

sv = v t_s t_e

Usage:

- Action effects
- Initial facts
- Timed initial literals (e.g. satellite visible during [120,420])

Effect tokens

Value v of a state variable sv over a temporal interval $[t_s, t_e]$

 $egin{array}{cc} {
m sv} = {
m v} \ t_s & t_e \end{array}$

Usage:

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- Initial facts
- Timed initial literals (e.g. satellite visible during [120,420])

Condition tokens

State variable sv must have value v over $[t_s, t_e]$ but it has not been established yet.

$$sv = v$$

Usage:

• Action's condition

te

- Goals
- Temporally extended goals

pick(r: robot, o: object, l: loc) start: t_s end: t_e duration: $4 (= t_e - t_s)$ conditions: $[t_s, t_e] \operatorname{at}(r) = I$ $[t_s - 1, t_e - 1] \operatorname{at}(o) = I$ effects: $[t_e - 1, t'] \operatorname{at}(o) = \operatorname{grip}$ $[t_s, t_e] \operatorname{busy}(r) = \operatorname{true}$



t': artificial, unbounded, timepoint to reason about the end of the effect
Two distinct tokens

$$\begin{array}{c} \mathbf{s}\mathbf{v}^{1} = \mathbf{v}^{1} \\ t_{s}^{1} & t_{e}^{1} \end{array} \qquad \begin{array}{c} \mathbf{s}\mathbf{v}^{2} = \mathbf{v}^{2} \\ t_{s}^{2} & t_{e}^{2} \end{array}$$

should not concurrently impose a value to the same variable

$$sv^1
eq sv^2$$
 \lor $t_e^1 < t_s^2$ \lor $t_e^2 < t_s^1$

A condition token

$$sv^1 = v^1$$

 t^1_s t^1_e

should be established by an effect token

$$sv^2 = v^2$$
$$t_s^2 t_e^2$$

$$\begin{array}{ccc} & \text{such that} \\ sv^1 = sv^2 & \wedge & v^1 = v^2 & \wedge & t_s^2 \leq t_s^1 \leq t_e^1 \leq t_e^2 \end{array}$$

- Start from a empty plan, containing only the tokens representing the initial facts ands goals
- Such partial plan is flawed: it might contains violations of the consistency rules
 - unsupported condition
 - conflicting assignment
- PSP algorithm: refines the partial plan until no flaw remains

Partial plan:

- Set of actions
- Set of timelines and associated tokens
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- 6. return **PSP(pplan')**

































PSP generates a search tree which can be explored:

- In Depth-First (common option among timeline-based planners)
- Through heuristic search $(A^*, A\epsilon, weighted A^*)$

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Choices to make:

- Which flaw to fix?
 - (often) with least resolvers (minimize branching factor)
- Which partial plan (A^*) or resolver (DFS) ?
 - Domain specific (common in timeline-based)
 - Least-commitment (IxTeT)
 - Distance to consistency [Bernardini and Smith, 2008, Bit-Monnot et al., 2020]

Lifted planning

- Action parameters are variables whose value are constrained
- The STN is complemented with a binding constraint network that maintain the possible values of parameters.

Planners: FAPE, IxTeT, Europa, LCP

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Other CSP-based extensions:

- Resource reasoning [Laborie, 2003]
- Spatial reasoning [Mansouri and Pecora, 2014]



Temporal Planning What? (which actions?) How? (which parameters?) When? Scheduling

Temporal Planning { What? (which actions?) How? (which parameters?) When? } Scheduling

- Mature and efficient tooling in *Operations Research* and *Constraint Programming* communities
- Many (most?) planning problems are naturally handle as scheduling.

A good entry point: CPOptimizer [Laborie et al., 2018]

Conclusions
Take-away message

Summary

- Temporal Planning = Deciding what to do + deciding when to do it
- State-oriented techniques
 - Strong heuristic guidance
 - Risk of temporal over-commitment
 - Harder to support complex temporal constraints
- Time-oriented techniques
 - Harder to provide heuristic guidance
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Temporal planning is an active area of research

- Continuous change: resources changing continuously
- Temporal uncertainty
- Combination of temporal planning with other extensions of classical planning:
 - Optimal temporal planning
 - Temporal planning with trajectory preferences

Thank you!

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Backup Slides

- Timeline-based planning over discrete time is EXPSPACE-complete, NEXPTIME-complete if plan horizon is known in advance (see [Gigante et al., 2017])
- *Qualitative* timeline-based planning over discrete time is PSPACE-complete (see [Della Monica et al., 2020])
- Timeline-based planning over dense time is undecidable, with many decidable fragments ranging from EXPSPACE- to NP-complete (see [Bozzelli et al., 2020])
- Timeline-based games (i.e. playing against the nondeterministic environment) are 2EXPTIME-complete (see [Gigante et al., 2020b])