Towards a hierarchical modelling approach for planning aircraft tail assignment and predictive maintenance

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Abstract
Aircraft equipment health monitoring system plays a promising role for airlines operation cost reduction, as it can be exploited to perform predictive maintenance. In this vein, a hierarchical sequential decision making model is proposed to plan predictive maintenance. It combines linear optimization for routing assignment and MDP planning to handle maintenance actions based on the stochastic evolution of health indicators. This entangled model should reduce planning time while ensuring a cost-efficient policy.

Introduction
Aircraft connectivity plays a promising role in operation cost reduction for airlines. Thanks to data collection of maintainable systems some equipment can be monitored, and unscheduled maintenance can be triggered by the evolution of the related health indicator (Wang, Chu, and Wu 2007). Note the visibility of upcoming failures can help to redefine the maintenance planning and reduce Aircraft On Ground (AOG) risk. In this context, predictive maintenance (Mobley 2002) aims to consider the real time values of health indicators, as well as their likely evolution, to plan unscheduled maintenance. However, specific aircraft routing is necessary to be able to perform maintenance in some cases. Thus, a planning model that controls routing allocation and maintenance actions could efficiently reduce operation costs.

Most of known approaches (Kelly 2006; Nicolai and Dekker 2008; Sriram and Haghani 2003) are based on linear formulations and only considers the planning of maintenance checks necessary after a given number of flight hours. However, such approaches hardly consider uncertainty, possibly related with health equipment evolution. A promising approach to handle predictive maintenance planning problems could be to consider a stochastic evolution of the health indicator, and to apply probabilistic decision models such as Markov Decision Processes (MDP) (Mausam and Kolobov 2012) for fleet predictive maintenance planning.

In this context, this paper proposes a hierarchical model that combines strengths of stochastic planning and linear optimization to handle the predictive maintenance problem.

The idea is to reduce the complexity of the planning problem tackled by the MDP solving algorithm by delegating the resolution of route assignment sub-problem to a linear program. The use a constrained formulation to solve the single step fleet allocation sub-problem, would result into aircrafts locations describing state variables in the MDP. Then, the MDP would choose the maintenance actions depending on the future slots and stochastic evolution of health indicators. It is expected that by decomposing the problem in this way, the dimensionality of MDP problem decreases, reducing planning time while ensuring a cost-efficient policy.

Overview of frameworks
Markov Decision Process
A finite-horizon Markov Decision Process (MDP) can be defined as a tuple $(S, A, D, P, C)$, such that $S$ is a finite set of states, $A$ is a finite set of actions, $D$ the decision steps set, with $|D| = T_{\text{max}}$ defining the time horizon, $P(s'|s, a, t)$ is the probability that action $a \in A$ in state $s \in S$ will lead to state $s' \in S$ at time step $t \in D$, $C(s', s, a)$ is the immediate reward received after applying action $a \in A$ in state $s \in S$. The goal of solving a finite horizon MDP is to find a deterministic Markov policy such as $\pi : S \times D \rightarrow A$ that minimizes the total expected cost for a given sequence of decision steps. The expected total gain is usually defined as:

\[
V^\pi(s) = \min_{\pi} \mathbb{E}\left[ \sum_{t=0}^{T_{\text{max}}-1} C(s_{t+1}, s_t, \pi(s_t)) \mid s = s_0 \right],
\]

where $\pi(s_t) = a_t$. Dynamic programming can be used to solve such a decision problem. For this, the equation above can be reformulated by breaking it down into a sequence of decision steps over time, resulting in the well-known Bellman equation. It is a fundamental result that leads to classical resolution algorithms as Value or Policy Iteration (Mausam and Kolobov 2012). Recently, performing algorithms have been proposed to solve MDPs (Keller and Helmert 2013; Kocsis and Szepesvári 2006). These algorithms are or trial heuristic search based either Monte Carlo tree search based.
Mixed Integer Linear Programming

Mixed Integer Linear Programming (MILP) consist in defining a linear objective function \( f : x \mapsto f(x) \) to be minimized with respect to some equality \( g(x) = c \) and inequality \( h(x) \leq d \) constraints that are also linear. Some specific formulations such as multi-commodity network flows (Sriram and Haghani 2003) may be useful in the context of sequential decision making. Existing solvers such as CPLEX or GUROBI are well suited.

Study case scenario

Let’s consider an horizon of days \( |D| \), and a fleet of \( n_p \) aircrafts flying through \( n_c \) cities. To each aircraft, we associate a stochastic process that defines the dynamic evolution of predictive maintenance indicators. These indicators vary following a Markov chain defined as follow:

\[
C_{ij} = \begin{cases} C_{ij}^a & \text{if in \text{sequence}} \\ C_{ij}^b & \text{if in \text{random}} \end{cases}
\]

Maintenance is mandatory because of AOG. The cost for a route as- 

Limitation even for recent solving algorithms. Thus, this en- 

Hierarchical Model

This paper proposes a hierarchical model that combines strengths of Markov Decision Processes and Linear optimization. The main idea is to delegate the resolution of route assignment sub-problem to the linear program, while ensuring to trigger maintenance actions given the stochastic evolution of indicators. In other words the action set handled by the MDL model is drastically reduced, when compared with a single MILP formulation, where a huge number of actions (combinations between aircrafts, airport and maintenance actions) should be evaluated constituting a strong limitation even for recent solving algorithms. Thus, this en- 

The state \( s_t \in S \) is composed by locations of \( n_p \) aircrafts and their corresponding health indicator values such that: 

\[
s_t = (loc_{i_1}^{\delta_1},...loc_{i_{n_p}}^{\delta_{n_p}},P_{i_1}^{\delta_{n_p}},...P_{i_{n_p}}^{\delta_{n_p}})
\]

The action \( a_t \in A \) can be described as a set of Boolean action variables defining which aircraft should proceed to maintenance, such that: \( a_t = (m_{i_1},...,m_{i_{n_p}}) \)

The cost function is defined as a linear objective function: 

\[
C(s_t,a_t) = \min_{i,j,k \in L} \sum_{i,j,k \in L} (C_{ijk} + C_{m,k}^{a}m_{i})x_{i,j,k}
\]

The MILP decision variables are the set of \( x_{i,j,k} \) that correspond for routing aircraft \( i \) from city \( j \) to city \( k \), and the predefined maintenance actions \( m_{i} \). And, \( L \) is the set of arcs defining routes to be done between two locations. When a maintenance \( m_{i} \) is triggered for aircraft \( i \), a constraint is added in order to ensure that aircrafts start theirs flights from their initial positions \( loc_{i} \).

The cost function is defined as a linear objective function:

\[
\min_\pi E \left[ \sum_{t=0}^{T_{\text{max}}-1} \sum_{i,j,k \in L} C(s_t, \pi^F(s_t,s_{t+1})) \right] \leq
\]

\[
V^\pi^F(s_0) = \min_\pi \left[ \sum_{t=0}^{T_{\text{max}}-1} \min_{i,j,k \in L} \left( C_{ij}^m + C_{m,k}^{a}m_{i} \right) x_{i,j,k} \right] + C_{ij}^m
\]

Despite of this tight bound, alternating MILP solving to de- 

Conclusion and future work

The hierarchical model that we propose is an opportunity to bridge the gap between existing linear formulations of airline operations planning processes with considerations on aircraft health indicators predictions. Experiments are needed to evaluate the proposed framework and to assess policy efficiency.

References

Keller, T., and Helmer, M. 2013. Trial-based heuristic tree search for finite horizon mdps. In ICAPS.


