Simple epistemic planning problem: a unification approach

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Abstract. In this paper, we introduced simple epistemic planning problems and we show how to solve them by using unification techniques.

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1 Introduction

In this paper, we consider Simple Epistemic Planning Problems (SEPP) of the form $(A, B, \pi(\sharp))$ where A and B are epistemic formulas and $\pi(\sharp)$ is a patron, $\sharp$ being a new atomic symbol. Patrons are expressions of the form $\sharp!$, $\Box_a\sharp!$, $\frac{1}{2}(a, \sharp)$, $\frac{1}{2}(a, \Box_b\sharp)$, $(a, \sharp)$ and $(a, \Box_b\sharp)$ where a and b are agents. They correspond to actions like public announcements ($\sharp!$, $\Box_a\sharp!$), semi-private announcements ($\frac{1}{2}(a, \sharp)$, $\frac{1}{2}(a, \Box_b\sharp)$) and fully-private announcements ($(a, \sharp)$, $(a, \Box_b\sharp)$). To solve the SEPP $(A, B, \pi(\sharp))$ is to determine if there exists an epistemic formula C such that $\models A \to \langle\pi(C)\rangle B$, $\pi(C)$ being the actions obtained from $\pi(\sharp)$ and C by replacing the unique occurrence of $\sharp$ by C.
In this paper, we show how to solve SEPP. In particular, we associate to an arbitrary given SEPP $(A, B, \pi(\sharp))$, a necessary and sufficient condition for the existence of a solution. Moreover, when such a solution exists, we construct a most general one.

2 Background of DEL

2.1 Syntax of Epistemic Logic

Let P be a countable set of atoms, and AGT a finite set of agents.

Definition 1. *(Language of epistemic logic) The language of multi-agent epistemic logic, $\mathcal{L}_K$ is defined as follows.*

$$A ::= p \mid \bot \mid \neg A \mid (A \wedge A) \mid \Box_a A$$

where $\Box_a A$ is read "agent a knows A". We use $\Diamond_a A ::= \neg\Box_a\neg A$.

2.2 Semantics of Epistemic Logic

Epistemic models are Kripke models based on equivalence relations.

Definition 2. *An epistemic model is a structure $M = (W, \sim, V)$, where*

- *W is non-empty set of worlds.*
- *$\sim$: $a \in AGT \mapsto \sim_a \subseteq W \times W$ is a function associating to each agent $a \in AGT$ an equivalence relation $\sim_a$ on W.*
- *$V : P \to 2^W$ is a valuation function on W.*

For any $w \in W$, the pair $s = (M, w)$ is called an epistemic state and we usually use M, w instead of (M, w).

Let $M = (W, \sim, V)$ be an epistemic model and $w \in W$. We define the satisfiability of a formula A in M, w, (in symbols $M, w \vDash A$) as follows:

- $M, w \vDash p$ iff $w \in V(p)$,
- $M, w \nvDash \bot$,
- $M, w \vDash \neg A$ iff $M, w \nvDash A$,
- $M, w \vDash A \wedge B$ iff $M, w \vDash A$, and $M, w \vDash B$,
- $M, w \vDash \Box_a A$ iff for all $w' \in W$, if $w \sim_a w'$ then $M, w' \vDash A$.
  As a result,
- $M, w \vDash \Diamond_a A$ iff there exists $w' \in W$ such that $w \sim_a w'$ and $M, w' \vDash A$

2.3 Action Models and Epistemic Actions

Action models are finite relational structures that can also be seen as syntactic objects [3, 8].

Definition 3. *(Action model) An action model is a structure $\boldsymbol{M} = (\boldsymbol{S}, \sim, \boldsymbol{pre})$ such that*

- *$\boldsymbol{S}$ is a non-empty and finite set of actions.*
- *for all $a \in AGT$, $\sim_a$ is an binary relation on $\boldsymbol{S}$.*
- *$\boldsymbol{pre} : \boldsymbol{S} \to \mathcal{L}_K$ is a function that assigns a precondition $\boldsymbol{pre(s)} \in \mathcal{L}_K$ to each $\boldsymbol{s} \in \boldsymbol{S}$.*

For any $\boldsymbol{s} \in \boldsymbol{S}$, the pair $\alpha = (\boldsymbol{M}, \boldsymbol{s})$ is called an epistemic action.

In this paper, we will consider specific epistemic actions like public announcements, semi-private announcements and private announcements.
Public announcements constitute a specific kind of action models [3, 8, 12]. The public announcement of $A \in \mathcal{L}_K$ is the action model $\mathbf{M} = (\mathbf{S}, \sim, \mathbf{pre})$ where

- $\mathbf{S} = \{\mathbf{s}\}$,
- for all $a \in AGT$, $\sim_a = \{(\mathbf{s}, \mathbf{s})\}$,
- $\mathbf{pre}(\mathbf{s}) = A$.

We will denote the epistemic action $(\mathbf{M}, \mathbf{s})$ by the notation $A!$ and it is read "all agents receive the message A as a public announcement".
Semi-private announcements constitute another specific kind of action models [4–6, 10, 11]. The semi-private announcement of $A \in \mathcal{L}_K$ to agent $a \in AGT$ is the action model $\mathbf{M} = (\mathbf{S}, \sim, \mathbf{pre})$ where

- $\mathbf{S} = \{\mathbf{s}, \mathbf{t}\}$,
- for all $b \in AGT \backslash \{a\}$, $\sim_b = \{(\mathbf{s}, \mathbf{s}), (\mathbf{t}, \mathbf{t}), (\mathbf{t}, \mathbf{s}), (\mathbf{s}, \mathbf{t})\}$ and $\sim_a = \{(\mathbf{s}, \mathbf{s}), (\mathbf{t}, \mathbf{t})\}$,
- $\mathbf{pre(s)} = A$ and $\mathbf{pre(t)} = \neg A$.

We use the notation $\frac{1}{2}(a, A)$ for the epistemic action $(\mathbf{M}, \mathbf{s})$ and it is read "agent a received the message A as semi-private announcement".
Fully-private announcements constitute another specific kind of action models [4, 8, 9].
The private announcement of $A \in \mathcal{L}_K$ to agent $a \in AGT$ is the action model $\mathbf{M} = (\mathbf{S}, \sim, \mathbf{pre})$ where

- $\mathbf{S} = \{\mathbf{s}, \mathbf{t}\}$,
- for all $b \in AGT \backslash \{a\}$, $\sim_b = \{(\mathbf{s}, \mathbf{s}), (\mathbf{t}, \mathbf{t}), (\mathbf{s}, \mathbf{t})\}$ and $\sim_a = \{(\mathbf{s}, \mathbf{s}), (\mathbf{t}, \mathbf{t})\}$,
- $\mathbf{pre(s)} = A$ and $\mathbf{pre(t)} = \top$.

We use notation (a, A) for the epistemic action $(\mathbf{M}, \mathbf{s})$ and it is read "agent a received the message A privately".

2.4 Update Product

Execution of epistemic actions may change the world and also agents' information. It is formalised by the update product of an epistemic model and an action model.

Definition 4. *(Update product) The update product of epistemic model $M = (W, \sim, V)$ with action model $\boldsymbol{M} = (\boldsymbol{S}, \sim, \boldsymbol{pre})$ is the epistemic model $M \otimes \boldsymbol{M} = (S', \sim', V')$ such that*

- $S' = \{(w, \boldsymbol{s}) \mid w \in W, \boldsymbol{s} \in \boldsymbol{S}$ *and* $M, w \vDash \boldsymbol{pre(s)}\}$,
- $(w, \boldsymbol{s}) \sim'_a (v, \boldsymbol{t})$ *iff* $w \sim_a v$ *and* $\boldsymbol{s} \sim_a \boldsymbol{t}$,
- $(w, \boldsymbol{s}) \in V'(p)$ *iff* $w \in V(p)$.

Of course, this update product exists only if there exists $w \in W$ and there exists $\mathbf{s} \in \mathbf{S}$ such that $M, w \vDash \mathbf{pre(s)}$.

2.5 Syntax and Semantics of Action Model Logic

Definition 5. *(Language of Action Model Logic) The language $\mathcal{L}_{AM}$ of Action Model Logic is defined as follows:*

$$A :: p \mid \bot \mid \neg A \mid (A \wedge A) \mid \Box_a A \mid [\boldsymbol{M}, \boldsymbol{s}]A$$

where $[\boldsymbol{M},\boldsymbol{s}]A$ is read "if action $(\boldsymbol{M},\boldsymbol{s})$ is executable, then after its execution, A holds". We use $\langle \boldsymbol{M},\boldsymbol{s}\rangle A ::= \neg[\boldsymbol{M},\boldsymbol{s}]\neg A$.

Definition 6. *Given epistemic state (M,w) with $M=(W,\sim,V)$. We define the satisfiability of a formula A in M,w (in symbols $M,w \vDash A$ as follows:)*

- $M,w \vDash p$ *iff* $w \in V(p)$
- $M,w \vDash \neg A$ *iff* $M,w \nvDash A$
- $M,w \vDash A \wedge B$ *iff* $M,w \vDash A$ *and* $M,w \vDash B$
- $M,w \vDash \Box_a A$ *iff for all* $w' \in W$, *if* $w \sim_a w'$ *then* $M,w' \vDash A$
- $M,w \vDash [\boldsymbol{M},\boldsymbol{s}]A$ *iff if* $M,w \vDash$ ***pre(s)*** *then* $(M \otimes \boldsymbol{M},(w,\boldsymbol{s})) \vDash A$

As a result,

- $M,w \vDash \langle \mathbf{M},\mathbf{s}\rangle A$ iff $M,w \vDash \mathbf{pre(s)}$ and $(M \otimes \mathbf{M},(w,\mathbf{s})) \vDash A$.

A formula A is valid (in symbol $\vDash A$) if $M,w \vDash A$ for any epistemic state M,w.

2.6 Some Validities

The following formulas are valid and will be used later in section 4 for solving simple epistemic planning problem.

$\Box_a A \to A$	(truth)
$\Box_a A \to \Box_a\Box_a A$	positive introspection
$\neg\Box_a A \to\to \Box_a\neg\Box_a A$	negative introspection
$[\mathbf{M},\mathbf{s}]p \leftrightarrow (\mathbf{pre(s)} \to p)$	atomic permanence
$[\mathbf{M},\mathbf{s}]\neg A \leftrightarrow (\mathbf{pre(s)} \to \neg[\mathbf{M},\ \mathbf{s}]A)$	action and negation
$[\mathbf{M},\mathbf{s}](A \wedge B) \leftrightarrow [\mathbf{M},\mathbf{s}]A \wedge [\mathbf{M},\mathbf{s}]B$	action and conjunction
$[\mathbf{M},\mathbf{s}]\Box_a A \leftrightarrow (\mathbf{pre(s)} \to \bigwedge_{\mathbf{s}\sim_a\mathbf{t}} \Box_a[\mathbf{M},\ \mathbf{t}]A)$	action and knowledge

As is well-known, the axiom for action and knowledge can be written as follows in the case of public, semi-private and fully-private announcements:

- $[A!]\Box_a B \leftrightarrow (A \to \Box_a[A!]B)$,
- $[\frac{1}{2}(a,A)]\Box_a B \leftrightarrow (A \to \Box_a[(a,A)]B)$,
- $[\frac{1}{2}(a,A)]\Box_b B \leftrightarrow (A \to \Box_b[(a,A)]B \wedge \Box_b[(a,\neg A)]B)$ where $a \neq b$.
- $[(a,A)]\Box_a B \leftrightarrow (A \to \Box_a[(a,A)]B)$,
- $[(a,A)]\Box_b B \leftrightarrow (A \to \Box_b B)$ where $a \neq b$.

Later in Section 4, we will also use the following admissible rules:

$$\frac{D \to \Box_a E}{\Diamond_a D \to E} \qquad\qquad \frac{\Diamond_a D \to E}{D \to \Box_a E}$$

3 Epistemic Planning Problem

An epistemic planning problem $T=(s_0,E,A_g)$ consists of: a finite epistemic state $s_0=(M,w)$ called the initial state; a finite set of epistemic actions E; a goal formula $A_g \in \mathcal{L}_K$. A sequence $(\alpha_1,...,\alpha_n)$ of actions from E is a solution to the epistemic planning problem T if $M,w \vDash \langle\alpha_1\rangle...\langle\alpha_n\rangle A_g$.

In its full generality, epistemic planning problem is undecidable even if the modal depth of preconditions is bounded by 2 [7]. In our setting, the considered epistemic actions will be public announcements, semi-private announcements and fully-private announcements. Since sequences of such actions can be written as a unique action, we introduced the following variant of epistemic planning.
Let $\sharp$ be a new symbol representing an unknown formula. Let $\mathcal{L}_K^\sharp$ be $\mathcal{L}_K$ extended by the new symbol $\sharp$. In $\mathcal{L}_K^\sharp$, the symbol $\sharp$ is considered as an atom. Patrons (denoted π, π' and etc) are expressions of the following forms:

- $\sharp!$: a public announcement saying that $\sharp$ holds.
- $\Box_a\sharp!$: a public announcement saying that agent a knows that $\sharp$ holds.
- $\frac{1}{2}(a, \sharp)$: a semi-private announcement to agent a saying that $\sharp$ holds.
- $\frac{1}{2}(a, \Box_b\sharp)$: a semi-private announcement to agent a saying that agent b knows that $\sharp$ holds.
- $(a, \sharp)$: a fully-private announcement to agent a saying that $\sharp$ holds.
- $(a, \Box_b\sharp)$: a fully-private announcement to agent a saying that b knows that $\sharp$ holds.

Note that each patron π contains exactly one occurrence of the new symbol $\sharp$. Moreover, for all $A \in \mathcal{L}_K$, let $\pi(A)$ be the expression obtained after replacing by A the unique occurrence of $\sharp$ in π. Obviously, $\pi(A)$ is an epistemic action. Simple epistemic planning problems are defined as follows.

Definition 7. *A simple epistemic planning problem (SEPP) is a triple (A, B, π) where $A, B \in \mathcal{L}_K$ and π is a patron. A formula $C \in \mathcal{L}_K^\sharp$ is a solution to the SEPP (A, B, π) if $\models A \to \langle\pi(C)\rangle B$.*
A formula $C \in \mathcal{L}_K^\sharp$ is a most general solution to (A, B, π) if $\models A \to \langle\pi(C)\rangle B$ and for all solutions D to (A, B, π), D is equivalent to an instance of C.

Not only, we will interest in the problem of solving a given SEPP (A, B, π), but we will also interest in the problem of finding its most general solution.

Example 1. Consider the SEPP $(A, B, \sharp!)$ where $A = \Box_1 p$ and $B = \Box_2 p$. It corresponds to the formula $\Box_1 p \to \langle\sharp!\rangle\Box_2 p$. To solve this SEPP is to find a public announcement C such that in any epistemic state, C holds each time the agent 1 knows p and after announcing C, the agent 2 knows p.
In this case $C = \Box_1 p$ is a solution since if agent 1 knows p then, after publicly announcing $\Box_1 p$ then the agent 2 knows p.

4 SEPP with public announcements

Let us see how to solve a SEPP of the form $(A, B, \sharp!)$. This SEPP corresponds to the formula $A \to \langle\sharp!\rangle B$. To solve it is to find $C \in \mathcal{L}_K^\sharp$ such that $\models A \to \langle C!\rangle B$. Let $P(\sharp) = A \to \langle\sharp!\rangle B$. First, we will use the validities and admissible rules of Section 2.6 to obtain a formula $P_1(\sharp) \in \mathcal{L}_K^\sharp$ which has the same solutions as $P(\sharp)$ and for which it seems easier to compute a most general solution. Then considering $P_1(\sharp)$, we will find a necessary and sufficient condition for the solvability of $P_1(\sharp)$. Finally, assuming this necessary and sufficient condition holds, we will construct a most general solution of $P_1(\sharp)$.

Lemma 1. *Let $P(\sharp) = A \to \langle \sharp ! \rangle (\Box_a B \wedge \Diamond_b C)$ where B, C are Boolean formulas, $A \in \mathcal{L}_K$ and $a, b \in AGT$. Let $B' = (\Diamond_a A \to B)$. The following are equivalent:*

1. $\models A \to B' \wedge \Diamond_b (B' \wedge C)$.
2. *$P(\sharp)$ is solvable.*

Moreover, B' is a solution of $P(\sharp)$ in that case.

Proof. Notice that the formulas $A \to \langle \sharp ! \rangle (\Box_a B \wedge \Diamond_b C)$ and $(A \to \sharp) \wedge (A \to \Box_a [\sharp] B) \wedge (A \to \langle \sharp \rangle \Diamond_b C))$ are equivalent (remind that B and C are Boolean formulas). Moreover, by using inference rules, one can easily show that the formulas $(A \to \sharp) \wedge (A \to \Box_a [\sharp] B) \wedge (A \to \langle \sharp \rangle \Diamond_b C))$ and $P_1(\sharp) = (A \to \sharp) \wedge (\sharp \to (\Diamond_a A \to B)) \wedge (A \to \Diamond_b (\sharp \wedge C))$ have the same solutions as $P(\sharp)$.
$(1 \Rightarrow 2)$ Suppose $\models A \to B' \wedge \Diamond_b (B' \wedge C)$. Hence, obviously, B' is a solution of $P_1(\sharp)$.
$(2 \Rightarrow 1)$ Suppose $P_1(\sharp)$ has a solution D for some $D \in \mathcal{L}_K^{\sharp}$. Consequently, $\models (A \to D) \wedge (D \to (\Diamond_a A \to B)) \wedge (A \to \Diamond_b (D \wedge C))$. Hence, $\models A \to B' \wedge \Diamond_b (B' \wedge C)$.

Lemma 2. *Let $P(\sharp) = A \to \langle \sharp ! \rangle (\Box_a B \wedge \Diamond_b C)$ where B, C are Boolean formulas, $A \in \mathcal{L}_K$ and $a, b \in AGT$. Let $B' = (\Diamond_a A \to B)$. If $P(\sharp)$ is solvable then there exists a most general solution of $P(\sharp)$: the formula $D = (\Box_b P_1(\sharp) \wedge \sharp) \vee (\neg \Box_b P_1(\sharp) \wedge B')$ where $P_1(\sharp)$ is the formula introduced in the proof of Lemma 1*

Proof. Suppose $P(\sharp)$ is solvable. Hence, by Lemma 1, $\models A \to B' \wedge \Diamond_b (B' \wedge C)$. Moreover, B' is a solution of $P(\sharp)$. From the proof of Lemma 1, we know that $P_1(\sharp)$ has the same solution as $P(\sharp)$. We claim that

1. D is a solution of $P(\sharp)$,
2. for all solutions D' of $P(\sharp)$, D' is equivalent to an instance of D.

About the first claim, we have to show $\models P_1(D)$. Notice that $P_1(D)$ is the conjunction of the following formulas:

- $A \to (\Box_b P_1(\sharp) \wedge \sharp) \vee (\neg \Box_b P_1(\sharp) \wedge B')$,
- $(\Box_b P_1(\sharp) \wedge \sharp) \vee (\neg \Box_b P_1(\sharp) \wedge B') \to (\Diamond_a A \to B)$,
- $A \to \Diamond_b \Big(\big((\Box_b P_1(\sharp) \wedge \sharp) \vee (\neg \Box_b P_1(\sharp) \wedge B') \big) \wedge C \Big)$.

Since $\models A \to B'$, the first formula above is equivalent to $A \wedge \Box_b P_1(\sharp) \to \Box_b P_1(\sharp) \wedge \sharp$ which is obviously valid.
The validity of the second formula above is left as an exercise for the reader.
Since $\models A \to \Diamond_b (B' \wedge C)$, the third formula above is equivalent to $A \wedge \Box_b (B' \wedge C \to \Box_b P_1(\sharp)) \to \Diamond_b (\sharp \wedge C)$ which is obviously valid.
About the second claim, let D' be a solution of $P_1(\sharp)$. Hence, $\models P_1(D')$. Thus, $\models \big((\Box_b P_1(D') \wedge D') \vee (\neg \Box_b P_1(D') \wedge B') \big) \leftrightarrow D'$. Consequently, D' is equivalent to the instance of D obtained after having replaced each occurrence of $\sharp$ in D by D'.

Proposition 1. *Let* $P(\sharp) = A \to \langle \sharp ! \rangle \Big((\Box_{k_1} B_1 \wedge ... \wedge \Box_{k_m} B_m) \wedge (\Diamond_{l_1} C_1 \wedge ... \wedge \Diamond_{l_n} C_n) \Big)$ *where* B_i *and* C_j *are Boolean formulas for* $1 \leqslant i \leqslant m$ *and* $1 \leqslant j \leqslant n$ *and* $k_i, l_j \in AGT$. *Let* $B' = (\Diamond_{k_1} A \to B_1) \wedge ... \wedge (\Diamond_{k_m} A \to B_m)$. *The following are equivalent:*

1. $\models A \to B' \wedge \Diamond_{l_1}(C_1 \wedge B') \wedge ... \wedge \Diamond_{l_n}(C_n \wedge B')$.
2. $P(\sharp)$ *is solvable.*

Moreover, B' *is a solution of* $P(\sharp)$ *in that case.*

Proof. Similar to the proof of Lemma 1.

Proposition 2. *Let* $P(\sharp) = A \to \langle \sharp ! \rangle \Big((\Box_{k_1} B_1 \wedge ... \wedge \Box_{k_m} B_m) \wedge (\Diamond_{l_1} C_1 \wedge ... \wedge \Diamond_{l_n} C_n) \Big)$ *where* B_i *and* C_j *are Boolean formulas for* $1 \leqslant i \leqslant m$ *and* $1 \leqslant j \leqslant n$ *and* $k_i, l_j \in AGT$. *Let* $B' = (\Diamond_{k_1} A \to B_1) \wedge ... \wedge (\Diamond_{k_m} A \to B_m)$. *If* $P(\sharp)$ *is solvable then there exists a most general solution of* $P(\sharp)$*: the formula* $D = (\bigwedge_{j=1}^{n} \Box_{l_j} P_1(\sharp) \wedge \sharp) \vee (\neg(\bigwedge_{j=1}^{n} \Box_{l_j} P_1(\sharp)) \wedge B')$ *where*

$P_1(\sharp) = (A \to \sharp) \wedge \bigwedge_{i=1}^{m} (\sharp \to (\Diamond_{k_i} A \to B_i)) \bigwedge_{j=1}^{n} \Diamond_{l_j}(\sharp \wedge C_j)$

Proof. Similar to the proof of Lemma 2.

Proposition 3. *Let* $P(\sharp) = A \to \langle \Box_1 ! \sharp \rangle \Big((\Box_{k_1} B_1 \wedge ... \wedge \Box_{k_m} B_m) \wedge (\Diamond_{l_1} C_1 \wedge ... \wedge \Diamond_{l_n} C_n) \Big)$ *where* B_i *and* C_j *are Boolean formulas for* $1 \leqslant i \leqslant m$ *and* $1 \leqslant j \leqslant n$. *Let* $B' = (\Diamond_{k_1} A \to B_1) \wedge ... \wedge (\Diamond_{k_m} A \to B_m)$. *The following are equivalent*

1. $\models A \to \Box_1 B' \wedge \Diamond_{l_1}(C_1 \wedge \Box_1 B') \wedge ... \wedge \Diamond_{l_n}(C_n \wedge \Box_1 B')$
2. $P(\sharp)$ *is solvable.*

Moreover, B' *is a solution of* $P(\sharp)$ *in that case.*

Proof. The proof is similar to the proof of Lemma 1.

Proposition 4. $P(\sharp) = A \to \langle \Box_1 \sharp ! \rangle \Big((\Box_{k_1} B_1 \wedge ... \wedge \Box_{k_m} B_m) \wedge (\Diamond_{l_1} C_1 \wedge ... \wedge \Diamond_{l_n} C_n) \Big)$ *where* B_i *and* C_j *are Boolean formulas for* $1 \leqslant i \leqslant m$ *and* $1 \leqslant j \leqslant n$. *Let* $B' = (\Diamond_{k_1} A \to B_1) \wedge ... \wedge (\Diamond_{k_m} A \to B_m)$. *If* $P(\sharp)$ *is solvable then there exists a most general solution of* $P(\sharp)$*: the formula* $D = (\bigwedge_{j=1}^{n} \Box_1 \Box_{l_j} P_1(\sharp) \wedge \sharp) \vee (\neg(\bigwedge_{j=1}^{n} \Box_1 \Box_{l_j} P_1(\sharp)) \wedge F)$ *where* $P_1(\sharp) = (\Diamond_1 A \to \Box_1 \sharp) \wedge \bigwedge_{i=1}^{m} (\Box_1 \sharp \to (\Diamond_{k_i} A \to B_i)) \bigwedge_{j=1}^{n} \Diamond_{l_j}(\Box_1 \sharp \wedge C_j)$

Proof. The proof is similar to the proof of Lemma 2.

$A \to \langle \sharp \rangle B$	Necessary condition	solution F	mgs D
$A \to \langle \sharp \rangle (\bigwedge_{i=1}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=1}^{n} \Diamond_{l_j} C_j)$	$\models A \to B' \wedge \bigwedge_{j=1}^{n} \Diamond_{l_j} (C_j \wedge B')$	$F = B'$	$D = (\bigwedge_{j=1}^{n} \Box_{l_j} P_1(\sharp) \wedge \sharp) \vee (\neg(\bigwedge_{j=1}^{n} \Box_{l_j} P_1(\sharp)) \wedge F)$

$A \to \langle \Box \sharp \rangle B$	Necessary condition	solution F	mgs D
$A \to \langle \Box_1 \sharp \rangle (\bigwedge_{i=1}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=1}^{n} \Diamond_{l_j} C_j)$	$\models A \to \Box_1 B' \wedge \bigwedge_{j=1}^{n} \Diamond_{l_j} (C_j \wedge \Box_1 B')$	$F = \Box_1 B'$	$D = (\bigwedge_{j=1}^{n} \Box_1 \Box_{l_j} P_1(\sharp) \wedge \sharp) \vee (\neg(\bigwedge_{j=1}^{n} \Box_1 \Box_{l_j} P_1(\sharp)) \wedge F)$

Table 1. Simple epistemic planning problem with public announcements

$A \to \langle (1, \sharp) \rangle B$	Necessary condition	Solution F	mgs D
$A \to \langle (1, \sharp) \rangle (B \wedge \Box_1 B_1 \wedge \Diamond_1 C_1 \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	$\models A \to B \wedge B' \wedge \Diamond_1 (B' \wedge C_1) \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	B'	$D = (\Box_1 P_1(\sharp) \wedge \sharp) \vee (\neg \Box_1 P_1(\sharp) \wedge F)$

$A \to \langle (1, \Box \sharp) \rangle B$	Necessary condition	Solution F	mgs D
$A \to \langle (1, \Box_t \sharp) \rangle (B \wedge \Box_1 B_1 \wedge \Diamond_1 C_1 \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j)$	$\models A \to B \wedge \Box_t B' \wedge \Diamond_1 (\Box_t B' \wedge C_1) \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	B'	$D = (\Box_t \Box_1 P_1(\sharp) \wedge \sharp) \vee (\neg \Box_t \Box_1 P_1(\sharp) \wedge F)$

Table 2. Simple epistemic planning problem with private announcements

$A \to \langle \frac{1}{2}(1, \sharp) \rangle B$	Necessary condition	Solution F	mgs D
$A \to \langle \frac{1}{2}(1, \sharp) \rangle (B \wedge \Box_1 B_1 \wedge \Diamond_1 C_1 \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	$\models A \to B \wedge B' \wedge \Diamond_1 (B' \wedge C_1) \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	B'	$D = (\Box_1 P_1(\sharp) \wedge \sharp) \vee (\neg \Box_1 P_1(\sharp) \wedge F)$

$A \to \langle \frac{1}{2}(1, \Box \sharp) \rangle B$	Necessary condition	Solution F	mgs D
$A \to \langle \frac{1}{2}(1, \Box_t \sharp) \rangle (B \wedge \Box_1 B_1 \wedge \Diamond_1 C_1 \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j)$	$\models A \to B \wedge \Box_t B' \wedge \Diamond_1 (\Box_t B' \wedge C_1) \wedge \bigwedge_{i=2}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=2}^{n} \Diamond_{l_j} C_j$	B'	$D = (\Box_t \Box_1 P_1(\sharp) \wedge \sharp) \vee (\neg \Box_t \Box_1 P_1(\sharp) \wedge F)$

Table 3. Simple epistemic planning problem with semi-private announcements

5 Conclusion

In this paper, we introduced Simple Epistemic Planning Problem with public announcements. In this respect, we considered the associated formula, $A \to \langle \sharp ! \rangle B$ and $A \to \langle \Box \sharp ! \rangle B$ where $A, B \in \mathcal{L}_K$and found necessary and sufficient condition for existence of a solution when announcements are public announcements. The method mentioned for SEPP with public announcements is adaptable for SEPP with semi-private announcements, fully-private announcements and group announcements [13]. In the case of SEPP with group announcement there is an open problem for which we could not find a solution:

$$A \to \langle \Box_1 \sharp_1 \wedge ... \wedge \Box_n \sharp_n \rangle (\bigwedge_{i=1}^{m} \Box_{k_i} B_i \wedge \bigwedge_{j=1}^{n} \Diamond_{l_j} C_j).$$

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